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THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED
WITH THE CO-OPERATION OF THE

AMERICAN PHYSICAL SOCIETY

BY

EDWARD L. NICHOLS,
ERNEST MERRITT, and FREDERICK BEDELL

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MARCH, 1911

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THE

PHYSICAL REVIEW.

THE CONSTRUCTION OF STANDARD CELLS AND A CONSTANT TEMPERATURE BATH.

By G. A. Hulett.

FOR exact electrical measurements a known electromotive force is indispensable and at present we rely on standard cells. From our knowledge of the source of the energy of a voltaic cell we might expect that any combination made up of two metallic electrodes surrounded by solutions of their respective salts would give a reproducible electromotive force which would be a function of the temperature only, but experience shows that this is not the case even when the two electrolytes are not allowed to mix by diffusion. The difficulty seems to be in the inhomogeneity of the surface of a solid metal, since solid metallic surfaces give variable potentials even against solutions of their own salts while metals in the liquid state do show the desired constancy and reproducibility.

The two combinations which have most nearly answered the requirements of a standard are the Clark cell—mercury, mercurous sulphate, zinc sulphate, zinc amalgam—and the Weston standard cell which is the same as the Clark cell except that the zinc is replaced by cadmium throughout. In these cells the cathodes are liquid mercury and each anode amalgam is made up of two phases, a liquid amalgam and a solid amalgam. The composition of the liquid amalgam, which determines the potential of the electrode, depends on its temperature only, while the composition of the electrode as a whole may vary over a considerable range. All these electrodes function as liquids.

In the Clark cell the zinc amalgam is surrounded by a saturated solution of zinc sulphate with an excess of the solid ($ZnSO_47H_2O$) which furnishes a definite concentration of zinc ions in the electrolyte for each temperature. Since the electrode is made up of two metals, mercury as well as zinc, the electrolyte in equilibrium with it must also contain some mercury ions. The relation between the zinc and mercury ions in the electrolyte is expressed by the Nernst equation

$$\sqrt[n_1]{\frac{P_1}{p_1}} = \sqrt[n_2]{\frac{P_2}{p_2}},$$

where P_1 and p_1 refer to the solution pressure of zinc and osmotic pressure of the zinc ions and these letters with the subscript (2) refer to these magnitudes of mercury, while n_1 and n_2 represent the valences of the metals. In our case the concentration of the mercury ions in the electrolyte (p_2) is exceedingly small but there is evidence that these ions are actually present and that they come from the electrode with an equivalent deposition of zinc in the amalgam.[1] The positive pole or cathode of this cell is mercury surrounded by an electrolyte saturated with both zinc sulphate and mercurous sulphate with an excess of both salts in the crystalline state. Here again the mercury will not remain pure mercury but gains a small amount of zinc from the electrolyte, sufficient to satisfy the equilibrium conditions of the equation. The zinc sulphate has a concentration of 3.36 molecular weights to a liter and the mercurous sulphate one of only .00168 or a .056 per cent. solution while the concentration of the zinc in the mercury electrode is something of the order of 10^{-10}.

The cathode and anode legs of these cells represent two different systems each of which comes to equilibrium at a given temperature, but the cell as a whole is not a system in equilibrium and cannot be as long as it shows an E.M.F. The dissolved mercurous sulphate in the cathode leg slowly diffuses over to the anode amalgam where it is at once reduced to the equilibrium concentration which was already present before it arrived. This is the function of the excess of zinc in the anode, but since the concentration of the mercury in

[1] Crenshaw, Jour. Phys. Chem., 14, 158.

the cathode leg is small and the electrolyte a very viscous solution, the diffusion of mercury to the anode is a slow process and the life of the cell is very great. These same relations obtain in the Weston standard cell where the zinc is replaced by the cadmium and it would seem that these two combinations were ideal from the standpoint of constancy and reproducibility. The one drawback seems to be in the mercury salt. It is a characteristic of mercury salts to hydrolyze, that is, to interact with water and form a basic salt and acid, and mercurous sulphate is not an exception to this rule.[1] Sulphuric acid and sulphate solutions prevent or decrease this hydrolysis, and the hydrolysis of mercurous sulphate in a zinc sulphate solution has not been detected but it is surely present in a cadmium sulphate solution. Such a reaction need not detract from the value of the cell for equilibrium is established as soon as the basic salt and a definite acid concentration are present and then the potential is again a function of the temperature only. In the Clark cell the reaction has not been detected but in the Weston standard cell the hydrolysis takes place, although it is a very slow change which does not come to equilibrium.[2] However this change is generally small as well as slow and the cell has other qualities which make it valuable.

Materials.—Any amount of work may be done in purifying the materials used in constructing standard cells, but only a few foreign substances (in traces) measurably affect the E.M.F. of these standards and for our purpose it seems best to go only so far as to insure the desired reproducibility since the simpler the operations are the greater will be the agreement between the cells constructed at different times and by different investigators.

Mercury.—Pure redistilled mercury may be obtained from reliable dealers and it answers every purpose. In lieu of this, ordinary mercury is chemically purified and then distilled.[3]

Mercurous Sulphate.—The depolarizer is chiefly responsible for variations in the reproducibility and constancy of standard cells and the preparation of the mercurous sulphate and the making

[1]Zeit. Phys. Chem., 49, 493.
[2]PHYS. REV., 30, 648.
[3]PHYS. REV., 21, 388.

of the paste is still the most difficult and important part of the construction of these standards. In 1884 Lord Rayleigh called attention to this difficulty and the point was emphasized by Kahle[1] and later by Jaeger and Lindeck,[2] but the first explanation of the changes which caused the trouble was offered by Carhart and Hulett.[3] Evidence was given to show that the irregularities in the standard cells were due to the presence of a basic mercurous sulphate which had been formed in preparing the mercurous sulphate or in making the paste. It was also shown[4] that mercurous sulphate is stable in a sulphuric acid solution when the concentration of the acid is molecular (98 grams H_2SO_4 to a liter) or greater, but when the acid strength drops below this value the mercurous sulphate begins to hydrolyze with the formation of a difficultly soluble basic salt—$Hg_2(OH)_2.Hg_2SO_4$. After this information was obtained it was found possible to suggest conditions for preparing the mercurous sulphate and making the paste so as to exclude the basic salt and the cells made under these conditions did not require the usual aging but showed a constant value at once and were in exceptionally good agreement.[5] There are a number of methods for preparing mercurous sulphate and in such a way that it is possible to maintain the concentration of the sulphuric acid of molecular strength (or greater), at the point where the mercurous sulphate is forming and under such conditions only the normal mercurous sulphate appears. All such preparations give essentially the same value to standard cells.[6]

It is easy to prepare mercurous sulphate chemically and with sufficient sulphuric acid present to prevent hydrolysis, but there is the inclusion and isomorphism of nitric acid, nitrates or other substances to be considered and while the depolarizer made by these chemical methods does, with the proper precautions, give correct values to the standard cells, it is our experience that the electrolytic mercurous sulphate gives on the whole the most uniform and reproducible values.

[1]Wied. Ann., 51, 225.
[2]Zeit. Instk., 21, 33.
[3]Trans. Amer. Electrochem. Soc., 5, 63.
[4]Zeit. Phys. Chem., 49, 483.
[5]Trans. Amer. Electrochem. Soc., 5, 71.
[6]F. E. Smith, Phil. Trans., 207 A, 395.

The electrolytic method[1] has the distinct advantage that only mercury and moderately dilute sulphuric acid are used and the only foreign substance to be looked after is sulphuric acid which may be effectively removed as indicated below. We use an acid having a density of 1.15, made by pouring one volume of concentrated (density 1.84) sulphuric acid into six volumes of water. When this acid is electrolyzed between a mercury anode and a platinum cathode, mercurous sulphate is formed and goes into solution at the anode, but when the acid is saturated (.2 grm. Hg_2SO_4 to the liter) the solid mercurous sulphate appears and covers the anode. At the cathode, hydrogen is liberated and a little mercury but the efficiency is very great, forming about 9 grams of the salt per ampere hour. If the mercurous sulphate is allowed to collect on the anode to any great extent, a secondary reaction may take place but this is to be avoided and a stirrer has been employed to keep the mercurous sulphate in suspension in the electrolyte while the current was passing,[2] but only a limited amount of mercurous sulphate could be formed at a time. Lately we have improved the method by using an inner dish to hold the mercury and so arranged that the mercurous sulphate passed over the rim of the inner dish and collected in the space between it and the outer dish. The accompanying sketch, Fig. 1, shows the arrangement. A large deep crystallizing dish was nearly filled with the acid and in this was placed a somewhat smaller and very much lower crystallizing dish. Mercury was poured into the inner dish to a depth of about one centimeter and electrical connection was made through a stiff platinum wire fused into the end of a glass tube so that 4 or 5 cm. of the wire extended out into the mercury. The glass tube was bent to fit into the inner dish and over the edge of the outer dish as indicated in the figure. A short spiral of platinum wire was suspended in the sulphuric acid to serve as cathode, the platinum wires used were 0.5 mm. as a current of over two amperes was generally used with the mercury anode surface of 175 cm. (The inner dish has a diameter of 15 cm.) A motor-driven stirrer was

[1]Carhart and Hulett, Trans. Amer. Electrochem. Soc., V., 59. F. A. Wolff, Trans. Amer. Electrochem. Soc., V., 56, and M. Coste et M. Etaix, Bull. de la Soc. des Electriciens, 2 Serie, VIII., 1908.

[2]Trans. Amer. Electrochem. Soc., V., 63. PHYS. REV., 22, 334.

used and was made from a glass rod which was bent at a right angle and so arranged that the L part of the stirrer passed over and near to the surface of the mercury. This stirrer was held in a brass tube *bb* which also carried the pulley *p*. This brass tube turned in bronze bearings in the tube *hh* which was firmly held by clamps (not shown). It was found to be important to have the stirrer well made so that it worked smoothly and with certainty.

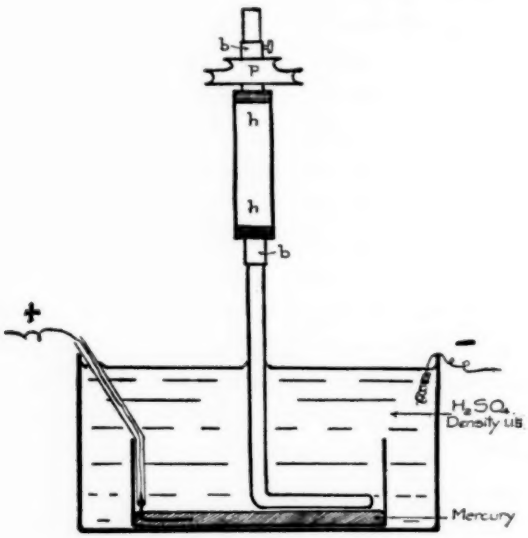

Fig. 1.

After the stirrer was running uniformly at a rate of some 200 revolutions per minute, the current was turned on. With fresh acid a skin formed on the surface of the mercury but by breaking and making the current several times or until the acid became saturated with the mercurous sulphate and the solid sulphate appeared throughout the acid, then there was no further trouble. The position of the stirrer was such that it did not unduly agitate the mercury. The mercurous sulphate formed was carried up by the rotating liquid and settled in the space between the two dishes and much more readily when the rotation of the liquid outside of the inner dish was retarded. This was easily accomplished by inserting glass plates edgewise down into the acid and allowing them to rest

on the edge of the inner dish. With a current density of from 1 to 2 amperes per 100 cm.^{2} of mercury anode surface the product obtained was gray, due to finely divided mercury. The presence of this finely divided mercury is an advantage in checking any tendency to oxidation. Generally 50 or 60 grams were prepared at a run and after the stirrer had been removed and the mercurous sulphate had settled, most of the acid was removed, the contents of the inner dish were poured into the outer dish and all well stirred for some time, then the acid, with the suspended sulphate, was poured into a clean dish and after the sulphate had settled the acid was returned to the mercury, stirred and again decanted and this was repeated until the mercury and mercurous sulphate were separated. The product was transferred to a glass stoppered bottle, covered with a little of the acid and kept in a dark place until needed. Some of the acid, diluted with two volumes of water, was brought into an ordinary washing bottle and the stream blown from this bottle aided materially in transferring the sulphate from one dish to another.

After this apparatus was once perfected and assembled it has been a simple matter to prepare the depolarizer whenever needed. Our experience is that this gray electrolytic mercurous sulphate, prepared as just described, is the most reliable and reproducible depolarizer for standard cells and that the grains are sufficiently large to avoid all effects of surface tension.[1]

The Flowing Anode Method.—Lately we have also used an interesting and very simple method for preparing electrolytic mercurous sulphate. A fine stream of mercury flowing from a funnel into the sulphuric acid is made the anode and no stirring is required. The accompanying sketch, Fig. 2, illustrates the apparatus. An ordinary funnel was used, the stem was warmed in a flame and drawn down to a capillary which was about 10 cm. in length and of such a diameter that 10 c.c. of mercury were delivered in about five minutes. An ordinary liter beaker glass was filled nearly full with the sulphuric acid (density 1.15) and the funnel adjusted so that the tip of the capillary was just under the surface of the acid. A short platinum spiral was hung in the acid

[1] PHYS. REV., 22, 326–328.

and served as cathode, mercury was poured into the funnel and it ran through the capillary and formed a spray as it entered the acid but when contact was made with the mercury in the funnel and a current of from 2 to 3 amperes passed through this mercury, the electrolyte, and out through the platinum spiral, the spray changed to a cylinder of flowing mercury which extended to the bottom of the beaker and looked like a wire. It was distinctly gray save for a short distance at the top which was bright mercury. M. Coste et M. Etaix[1] have used an alternating current in preparing mercurous sulphate. The flowing electrode method also permits us to use an alternating current and very simply. Two funnels were prepared with the same length of capillary stems and of the same diameter. These funnels dipped into the same beaker of sulphuric acid and the two streams of mercury were used as the electrodes, a 60-cycle alternating current was used and so regulated that there was no arcing between electrode and electrolyte and thus a current of about 5 amperes was used. The efficiency here is as great as with the direct current, about 9 grams of mercurous sulphate per ampere hour. With this flowing anode method the preparation is white, even with large current densities, and the sulphate is easily separated from the mercury by decanting it with the electrolyte. Cells made with these preparations as depolarizers show a slightly higher value than with the gray electrolytic previously described. The alternating current preparation seems to

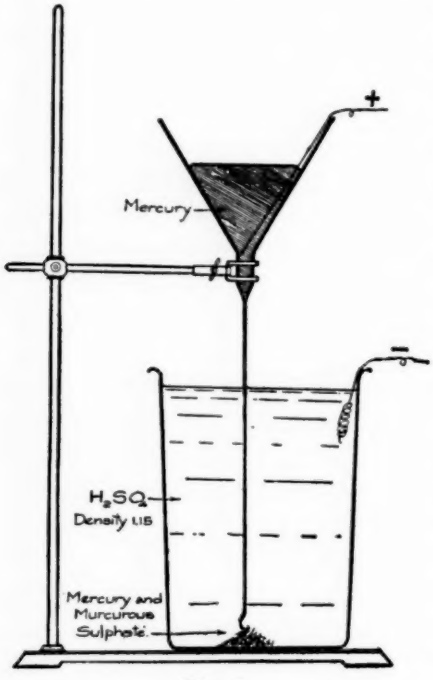

Fig. 2.

[1]Bull. de la Soc. des Electriciens, 2 Serie, VIII., 1908.

be the better of the two but they have not been tested for a sufficient length of time if we are concerned with the fifth decimal place in the E.M.F. of the cells.

The Paste.—The problem is to prepare a mixture of mercurous sulphate, zinc or cadmium sulphate and saturated solution, but to avoid the presence of sulphuric acid and basic mercurous sulphate. The only impurity in the electrolytic mercurous sulphate is the sulphuric acid in which it is made and preserved but any attempt to remove this acid by washing with water introduces the basic salt. Formerly we used alcohol which was subsequently removed by washing with the saturated zinc or cadmium sulphate solution, but we have found that this preliminary washing with alcohol is not necessary and that really only three washings with the saturated sulphate solution are necessary, when certain precautions are observed. A Gooch crucible, filtering apparatus and good suction were employed. A disk was cut from filter paper and fitted nicely to the bottom of the crucible and then the mercurous sulphate was transferred to the crucible and washed down with the acid. The layer of solid sulphate was about 15 mm. thick and the acid was quite completely removed by suction. By shaking off the drops which formed on the under side of the crucible it was determined when no more of the acid was being removed by suction. The inside of the crucible down to the sulphate was wiped with a bit of filter paper, to remove any drops of acid, and then the sulphate was washed directly with the saturated zinc or cadmium sulphate solution, using one to two cubic centimeters each time and taking care to remove this solution quite completely by suction before a second portion was added. It had been our practice to wash five or six times in this way but some experiments have shown that twc washings gave the same value to the cells as was obtained when the sulphate had been washed five or six times. Now we wash only three times with the saturated zinc or cadmium sulphate solution but with due attention to the details just given. The washed sulphate was quite compact and the top layer with any dust was easily scraped off, then the solid cake was removed by tapping the inverted crucible on a piece of filter paper. A large agate mortar was used in mixing the components of the paste. Zinc or

cadmium sulphate crystals, about equal in volume to the mercurous sulphate, were crushed but a few cubic centimeters of the saturated solution were first added to the crystals to prevent partial dehydration by the crushing.[1] After the crystals had been ground to a fine powder, the mercurous sulphate was added and all thoroughly mixed with enough of the saturated solution to make a thin paste and of such a consistency that it readily flowed from a 5-mm. tube. Unless the mercurous sulphate was gray a little mercury was ground up with the crystals and mixed with the paste. In transferring the paste to the cells a thin-walled glass tube 30 cm. long, 5 mm. in diameter was slightly contracted at the lower end and used as a pipette. By gentle suction, and stirring the paste with the end of the pipette, the paste was drawn up into the pipette and easily transferred to the cell and added until it covered the mercury electrode to a depth of about 15 mm. In all the operations of preparing the depolarizer and making the paste, direct sunlight or undue exposure to light was avoided. Excessive exposure to light darkens mercurous sulphate and the product then gives a higher value to the E.M.F. of the standard cells.

Zinc Sulphate. — $ZnSO_4 7H_2O$ is one of our more soluble salts. The solubility increases rapidly with the temperature and at 39° the solid phase changes to the hexahydrate which is to be avoided in making the Clark cells. E. Cohen gives the following relation between solubility and temperature; $L_t = 41.80 + .522t + .00496t^2$. From this we find that 100 c.c. of water dissolves:

At	$ZnSO_4$. Grams.	$ZnSO_4 7H_2O$. Grams.
0°	41.80	110.6
0°	41.80	110.6
15°	50.78	154.9
20°	54.22	167.6
25°	57.92	188.5
30°	60.92	207.3
39°	70.66	275.3

Zinc blend, from which this salt is obtained, generally contains cadmium and manganese and often lead and tin. The salt is

[1] W. F. Hildebrand, J. Amer. Chem. Soc., 30, 1120.

isomorphous with the other vitriols—iron, magnesium, cobalt' nickel and copper, but none of these metals, in the small amount usually found present in zinc sulphate, seems to affect the E.M.F. of the Clark cells. Sulphuric acid is to be excluded as .027 per cent. of this acid in the electrolyte of the cell reduces the E.M.F. measurably.[1] Zinc sulphate, as obtained in the trade, generally contains less than the theoretical amount of water of crystallization due to efflorescence, so it is best to recrystallize the salt and at room temperature in order to insure the heptahydrate. Starting with "chemically pure" zinc sulphate we made a saturated solution at room temperature, the solution showed a density of 1.45 and was brought into a flask with a handful of granulated zinc, warmed to 50° and frequently shaken for an hour or so in order to remove less positive metals than zinc and also any excess of acid. The solution was now filtered until perfectly clear. It was then poured into a large crystallizing dish, but not over a centimeter in depth. The dish was placed where the temperature was fairly constant and covered with filter paper. Due to the shallowness of the solution the crystals formed on the bottom of the dish and in a day or so a good crop of perfectly clear crystals one to two centimeters long were obtained. If a crust began to form on the solution or around the edges, a few drops of water were carefully run on to correct the difficulty and with little attention only clear well formed crystals were obtained. The solution was poured off and the crystals brought onto sheets of filter paper to absorb the excess of solution and then they were preserved in a well stoppered bottle to prevent efflorescence. A saturated solution is easily made from these crystals; the preceding table will indicate the amounts to use.

Zinc Amalgam.—Zinc is one of the more soluble metals. With an excess of zinc we have, for each temperature, a liquid amalgam of definite composition and in equilibrium with a solid phase which is pure zinc.

At	0°	the	liquid	amalgam	contains	1.35	per	cent. of zinc.
	15°	"	"	"	"	1.74	"	" "
	20°	"	"	"	"	1.99	"	" "
	25°	"	"	"	"	2.18	"	" "
	39°	"	"	"	"	2.85	"	" "

[1] PHYS. REV., 27, 352.

For our purpose it is seen that a 3 per cent. amalgam would serve every purpose since for the range of the Clark cell, such an amalgam is always two phase, but a 3 per cent. amalgam is rather too liquid while the 10 per cent. amalgam commonly used is too hard. We have found a 7 per cent. amalgam to be plastic at ordinary temperatures and most satisfactory. There seems to be no difference between the potentials of amalgams made from "chemically pure" zinc and mercury and amalgams made from the most highly purified metals so the "chemically pure" materials serve every purpose. Zinc dissolves but slowly in mercury and the amalgam oxidizes readily when hot but we have had no trouble with this amalgam since using the following method: A 2.5 cm. hole was cut in a piece of asbestos cardboard and an ordinary porcelain crucible was pushed into this hole so that it was about half way through, the board was placed on a tripod and the crucible was charged with about 7 grams of chemically pure zinc (weighed to .010 gram) and then thirteen times the weight of mercury was added. A small adjustable bunsen flame was placed under the crucible and the contents heated, but without stirring, until the zinc had all dissolved. This takes a temperature near to the boiling point of mercury and the crucible was covered with a little watch glass. The flame was now pushed a few centimeters to one side and when the crucible cooled down to about 100° the amalgam was readily transferred to the cells with a pipette. The pipette was made from a thin-walled glass tube 5 or 6 mm. in diameter with the tip drawn down in the flame to about 3 mm. The amalgam as prepared had only a slight coat, which was readily brushed to one side and the clean amalgam was sucked up into the pipette and transferred to the (warmed) anode leg of the cell without getting it on the sides of the glass. The amalgam was generally run in to a depth of about 10 mm. It soon cooled and solidified and then was ready for the zinc sulphate crystals and the saturated solution.

Cadmium Sulphate. — $CdSO_4 8/3 H_2O$ is a very soluble salt but is peculiar in that the *rate* of solution is exceptionally slow so that considerable attention is needed to prepare a saturated solution. The solubility changes only slightly with the temperature and the 8/3 hydrate is stable up to 74° where it changes to the monohydrate.

100 c.c. of water dissolve the following amounts at the indicated temperatures:

At	$CdSO_4$.	$CdSO_4 \cdot \frac{8}{3} H_2O$.
0°	75.5	112.5
15°	76.1	113.4
25°	76.8	114.7
40°	78.5	117.9
74°	78.6	133.8

Cadmium sulphate does not seem to be isomorphous with any known salt and consequently is obtained in a sufficiently pure state in the trade but one may obtain such beautiful crystals that it is worth while to recrystallize the salt. We dissolve a kilogram of the salt in a liter of water, using a mechanical stirrer, and filter the liquid into large crystallizing dishes. The depth of the liquid in the dishes is generally about one centimeter and the dishes are placed where the temperature is quite constant and covered with filterpaper. As the solution slowly evaporates perfect crystals appear on the bottom of the dish and most of them are water clear but some are cloudy, due to minute liquid inclusions. We have not found the cells made with the cloudy crystals to differ in E.M.F. from those made with the perfectly clear crystals; the cloudy crystals may of course be redissolved and recrystallized. Attention is to be given to the preparation of a saturated solution of cadmium sulphate on account of the slow rate of solution of these crystals; 100 c.c. of the saturated solution requires 75 c.c. of water and 86 grams of the crystals but a considerable excess of the crystals are to be used and stirred over night with a motor-driven stirrer.

Cadmium Amalgam. — Cadmium is the most soluble metal in mercury; the saturated amalgam contains the following percentages of cadmium:

At	Per Cent. Cadmium.
0°	2.5
15°	4.4
25°	5.6
35°	7.1

With an excess of cadmium the solid phase is not cadmium but an isomorphous mixture of cadmium and mercury.[1] The composition

[1] Bijl, Zeit. Phys. Chem., 41, 641.

of this solid phase also varies with the temperature but within certain limits the potential of this amalgam is definite and reproducible for a given temperature. In view of the great solubility of cadmium at ordinary temperatures it does not seem well to use less than 8 per cent. of cadmium in the amalgam. 12½ per cent. has been commonly used but Cohen and Krypt[1] have shown this amalgam to be unreliable below 14° and there is a question as to whether the solid phase of these amalgams are homogeneous and in equilibrium with the liquid phase when the amalgam is cooled from a higher to a lower temperature.[2] The question of equilibrium in this amalgam evidently needs further investigation; it is a much more complicated system than is the zinc amalgam.

We have used for some time a 10 per cent. amalgam for our cells and make it quite accurately and uniformly as follows: 99.00 grams of mercury were placed in a little crystallizing dish, 5 or 6 cm. in diameter, and 25.00 grams of the clear cadmium sulphate crystals (43.82 per cent. Cd) were placed on this mercury and then about 50 c.c. of distilled water was carefully added and made acid with a drop of sulphuric acid. A flat platinum spiral was so adjusted that the spiral was just beneath the surface of the water and contact was made with the mercury which was cathode. When 2 or 3 amperes were used the cadmium was deposited in the mercury about as fast as the sulphate dissolved, then when the crystals had all disappeared the current was increased to 4 or 5 amperes for half an hour. This insured a complete deposition of the cadmium and also liberated enough heat to melt the amalgam. The acid was finally removed by a pipette or syphon and at the same time distilled water was run in. This washing continued until the current dropped to zero. The amalgam may be left under water with the "current on" as long as one pleases as there is no possibility of loss or oxidation as long as the amalgam is cathode. When needed the amalgam was easily melted by removing the water and adding hot water and the hot water was finally removed quite completely by a pipette and then the liquid amalgam transferred

[1]Zeit. Phys. Chem., 65, 359.

[2]F. E. Smith, Phil. Mag., 19, 250. S. W. J. Smith, Phil. Mag., 20, 206. G. A. Hulett, Trans. Amer. Electrochochem. Soc., 15, 438.

to the cells as described under "zinc amalgams." These quantities suffice for 5 or 6 cells in all cases.

We have prepared cadmium sulphate and cadmium of the highest degree of purity,[1] also zinc sulphate and zinc[2] and constructed cells with these materials but the E.M.F. of these cells did not differ from the cells made with materials prepared as described above and this result is the one we would expect when we consider the factors controlling the potentials at the anodes and cathodes of these standard cells. For example a trace of iron sulphate could not affect the zinc amalgam nor measurably decrease the solubility of the zinc sulphate crystals, and the potential here depends on the ratio of the concentrations of the zinc in the amalgam to that in the electrolyte, nor would a trace of iron affect the mercury of the cathode or the solubility of mercurous sulphate. On the other hand a trace of basic mercurous sulphate would directly increase the concentration of the mercury at the cathode and since this concentration is small, the effect on the E.M.F. would be relatively large. Sulphuric acid also directly and measurably affects the concentration of the mercury in the catholyte and so the greatest attention was given to these points.

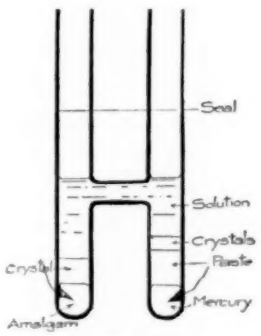

Fig. 3.

The Glass Parts.—The "H" cell proposed by Lord Rayleigh has proved to be the most practical form of cell. It is easily filled and allows the contents of each electrode to rapidly take up the temperature of the bath. We use thin-walled tubing about 10 mm. in diameter made into the form indicated in Fig. 3. The .2 mm. platinum wire leads are sealed through the glass so that all but the tip end is covered with a sheath of glass. The wire is first sealed in so that the end is only just through the glass and then, while the glass is still soft, the wire is pushed in about 5 mm. and covers itself with a sheath of glass leaving only the tip exposed. Wires sealed in this way give uniform contact with the mercury

[1]Jour. Amer. Chem. Soc., 30, 1808, 10.
[2]Jour. Phys. Chem., 14, 161–164.

or amalgams and are less liable to cause subsequent cracking of the glass especially when a fine wire is used.

The amalgam and mercury are introduced by a 5-mm. thin-walled pipette made of tubing some 30 cm. long with the lower end drawn down in the flame to about 3 mm. A good piece of rubber tubing attached to the upper end of this pipette makes it more convenient to manipulate. This pipette is also used for introducing the paste. The mercury, amalgam and paste are each from 10 to 15 mm. in depth. A few crystals are placed on top of the paste and on the amalgam a layer of crystals not over 10 mm. in depth. Only a small amount of the crystals are necessary for either the Clark or Weston cell, while a large excess may grow together and cause trouble. The cell is filled to the top of the cross tube with the saturated solution and then the glass parts are sealed off 2 or 3 cm. above the liquid by using two small blast flames which impinge on opposite sides of the point to be sealed. The part to be sealed is gently warmed at first to prevent cracking and after sealing, by properly heating a bit lower down, the expanding gas inside the cell will cause the seal to round out perfectly. The cells are arranged in supports and provided with lead wires so that they may be readily compared when in a bath.

The difficulties we have encountered with these cells are, the tendency of the glass parts to crack which is most pronounced in the amalgam leg of the Clark cells, and the fact that the contents of the cell are not accessible. This difficulty was recognized by Lord Rayleigh in recommending the cork seal.[1] It is often desirable to get at the contents of a cell to loosen a crust which has formed or to find a poor contact and while the glass seal may be broken it is seldom possible to reseal it without ruining the cell. The trouble with the cork seal has been that the cork was generally in contact with the liquid and so leaked sooner or later. We have used for some time a form of cell which avoids these difficulties; the glass part is the simplest possible (Fig. 4). Thin-walled tubes, 20 cm. long and 12 mm. in diameter, are closed at one end and blown out

[1]Phil. Trans., 176, 42, 1886. Cells were first hermetically sealed by Wright, Phil. Mag., 16, 25, 1883.

at the side about 5 cm. above the closed end. Two of these tubes are joined directly together giving the advantages of the test-tube form. No platinum wires are fused into this part of the cell. These cells are closed by corks which are 15 cm. above the liquid of the cell and are never wet and do not need to be covered with wax. These corks carry the long narrow tubes which inclose the contact wires and these may be removed and replaced at any time. This long form of cell is found to have many advantages in handling in the bath; several may be bound together in a very compact form and the contents of these cells most readily take on the bath temperature while the insulation is perfect in any kind of a bath liquid.

The details of the contact wire and protecting tubes are also shown in Fig. 4. A piece of tubing 3 mm. in diameter is softened in the flame and drawn down so that the part which is to pass through the contents of the cell is not over 2 mm. in diameter. This narrow part is about 6 cm. long while the total length is about 22 cm. A piece of .1 mm. platinum wire about 10 cm. long is soldered toa 20-cm. piece of silk-covered copper wire (no. 32). This wire is passed into the protecting tube until the platinum wire projects one or two millimeters from the narrow end of the tube and this end is then sealed in the flame. A bit of wax is run into the upper end to hold the wire securely. Six cells are made at one time and all bound together and then the negative or anode wires are twisted together and the exposed ends soldered so that only one contact need be made for the anodes of all six cells. The positive or cathode wires are scraped to remove the covering and then each one is wound about a small piece of millimeter copper wire which is forced into the warmed wax in the top of the tube leaving about a centimeter exposed and here a good contact is easily and rapidly made with a pinch connector. It is well to amalgamate the platinum tip of the contact wire just before it is inserted into

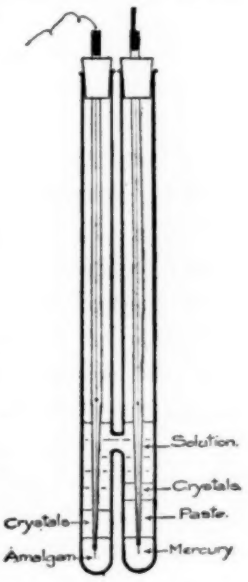

Fig. 4.

the contents of the cell. This is readily done by holding it for a few minutes in boiling mercury (in a test-tube), a suggestion of Mr. J. S. Laird of this laboratory. The numeral of the cell is always etched on the glass part and also written on a little pasteboard disk which is pierced with a small hole and put down over the lead wire and onto the cork so that it is easily read from above when the cell is in the bath. These cells are filled in the same way as the old form but instead of sealing the tops the contact wires with the protecting tubes and corks are inserted.[1]

In the old form of cell a crack was liable to develop at the point where the platinum lead wires passed through the glass. This was most noticeable in the amalgam leg of the Clark cell but would often happen in both legs of either cell. Sometimes the crack was hardly perceptible but sufficient to admit some of the bath oil in time while occasionally it was large enough to allow some of the liquid contents of the cell to escape and thus many cells were rendered worthless. The new form of cell entirely obviates this difficulty. Even if a crack develops where the platinum contact wire is sealed in the protecting tube no damage is done to the cell for this wire and tube is easily removed, repaired and replaced without injury to the cell. It frequently happens that the crystals over the amalgam cake and often a little gas forms between the amalgam and the liquid and crystals, sufficient to break the contact. By removing the contact wire and protecting tube, this gas escapes through the hole left by the tube and is replaced by the saturated solution and then the contact wire may again be inserted, or the crystals may be broken and stirred. This difficulty has been most noticeable in the Clark cells but also happened with some Westons.

The Electromotive Force of Standard Cells.—We have made both Clark and Weston cells at intervals since 1903 and according to the preceding specifications. A dozen Weston cells, made at one time, often agree among themselves to one part in 100,000 but in time the agreement is not so good, in a year or so the variations may be noticeable and when compared with a freshly made set of these cells it has been found that all the older cells have decreased

[1] Both the (H) and long form of cell have been satisfactorily made for us by Eimer & Amend, New York.

and some of them very noticeably. Clark cells have also been made at intervals during the last seven years and the agreement among themselves of any "set" of these cells is about the same as that of the Westons but the Clark cells made at different times are in better agreement and none of them have shown an E.M.F. decreasing with time such as is noticed in some of the Weston cells. A reason for this difference has been given in previous papers. It seems necessary therefore to make the Weston cells at intervals of about six months and reject those which fall to low values. We make both Clark and Weston cells at such intervals and the cells are all kept in a constant temperature bath which does not vary over .01° from 25.00°. Taken all together these two kind of cells with the aid of a thermostat, give us a standard which is independent of time and is reliable and reproducible to one or two parts in 100,000.

The value in absolute units of this standard of E.M.F. is of course not known to anything like this degree of accuracy but whenever the absolute value is more accurately determined, all work which has been based on this constant may be recalculated if necessary, so we are justified in giving the assumed value to the fifth decimal place.

We have for consideration the following determinations of the E.M.F. of the Clark cell in absolute units:

Rayleigh and Mrs. Sedgwick,	1884	1.4345 at 15°
Kahle,	1896	1.4322 " "
Carhart and Guthe,	1899	1.4333 " "
Ivanoff,	1900	1.4328 " "

The average is 1.4333 as the E.M.F. of the Clark cell at 15°, but these cells were all made in the old way and must have been about .0003 volt too high; allowing for this and calculating to 25° we get 1.42040. In 1906 Guthe,[1] working with standards constructed according to the preceding specifications and with a suitable thermostat, obtained the value 1.4204 for the Clark cell at 25° and 1.0184 for the Weston standard cell. Two determinations were made in 1908 of the value in absolute units of the Weston standard cell.

[1]Bull. Bureau of Standards, 2, 33, 1906.

Ayerton, Mather and Smith[1] obtained the figure 1.0183 at 17° and Janet, Laport and Jouaust,[2] found the value 1.01885 at 16°. Calculated to 25° these values are 1.01797 and 1.01847 respectively, but the results of these later determinations hardly justify a change in the values which have been used and it may be questioned whether the value in absolute units of our standard of electromotive force is known to better than one part in 2,000, or five parts in the fourth decimal place.

Assuming that the Clark cell is 1.4330 at 15° and using the customary temperature formula we have:

$$E_t = 1.4330 - .00119(t - 15) - .000007(t - 15)^2;$$

the value at 25° is 1.4204. Our cells numbering about 180 are maintained constantly at 25.00° and when we give this value (1.42040) to the Clark cells, the Weston standard cells, or at least those which have not decreased, are found to have the value 1.01840.

The Thermostat.—The E.M.F. of both the Clark and the Weston cells vary with the temperature. The Weston cell has only about one thirtieth of the temperature coefficient of the Clark cell and this was a decided advantage before thermostats were used but for electrical measurements which make any pretense to accuracy, an automatically controlled bath is indispensable for the Weston as well as for the Clark cell so the temperature coefficient is of no consequence. Our thermostat is reliable and constant to .01° and such variations indicate an uncertainty of about 1 in 100,000 in the E.M.F. of the Clark cell or within the variations of the cells themselves. The most reproducible temperature and the one easiest to maintain is 0° C., but the standard cells are very slow to come to their true value at this temperature, due probably to the slow rate of the reactions at low temperatures. The transition point of Glauber's salt is also available and an exceedingly well defined temperature (32.383), but it is not easy to maintain this temperature for the desired length of time. It is our experience that it takes a considerable time at a given temperature for *all* the cells of a set of standards to attain their true value. This can

[1]Philos. Trans., A, Vol. 207, 34.
[2]Bull. de la Soc. Electriciens, 1908.

be determined by daily observations when the cells are held at a constant temperature; we generally allow a week. It is therefore necessary to have an automatically controlled bath and 25° has been chosen as sufficiently above room temperature to allow of an exact control of the temperature and also moisture may be more readily excluded than at a lower temperature. Our thermostat is an electrically heated and controlled kerosene bath which keeps well within .01° for any length of time. It consists of a tank made from galvanized sheet iron, 50 × 65 cm. and 50 cm. deep which rests in a box on legs. The space between the box and the tank (6 cm.) is packed with excelsior and the top of the tank projects about 2 cm. above the top edge of the box. A cover, 100 × 100 cm., was provided and the center (50 × 65 cm.) cut out so that the top of the tank fits into this place and is flush with the top of the table part when assembled. Plate glass strips of convenient widths cover the tank proper.

About 100 liters of kerosene serve as the bath liquid and has been found to be very clean and easily kept dry by a dish of calcium chloride which is suspended in the upper part of the tank. A motor-driven stirrer was necessary and attention was given to having it well made. The stirrer shaft turns in a bearing which is held by a cross bar near the top of the tank and a second bearing is soldered to the bottom of the tank. This shaft bears a fan wheel which is close to the bottom of the tank with its fans set rather flat so as to force the bath liquid up from the bottom rather than to rotate it. At the top of this stirrer shaft is the pulley wheel, a few centimeters above the top of the bath.

The regulator and heater have received the most attention, the aim has been to get large surfaces well distributed in the bath liquid. Both the regulator and heater are supported by a frame about 10 cm. above the bottom of the tank. This frame was made of brass tubing, 40 × 55 cm., with two cross pieces and is supported by four legs. Fastened to the under side of this frame is the glass part of the regulator (Fig. 5), which contains the toluene and mercury and the expansion or contraction of these liquids makes or breaks a contact of the relay circuit and so controls the heating current. This regulator was made of 15-mm. tubing with walls

½ mm. thick. It is entirely closed save for the outlet through the U tube *a* which is sealed to the under side, extends down and then up ending in the 5 mm. upright tube *b*. The whole system was filled with toluene, except the U tube and the upright *b* which contains mercury. Air bubbles were rigidly excluded from this system. The top of the mercury column in the tube *b* makes contact with a platinum point and this surface is about 5 mm. in diameter, since it was found that a meniscus of this diameter was much better defined than the narrower ones commonly used while with the large volume of toluene well distributed in the bath liquid, the movement of this mercury surface responds rapidly and exactly to the slightest changes of the bath temperature.

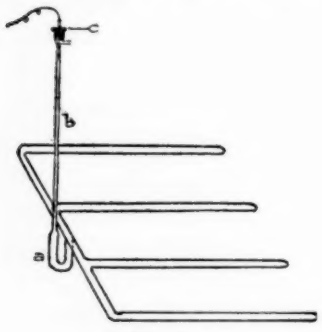

Fig. 5.

Sparking between the mercury and the platinum point causes the mercury to become "dirty," changes the meniscus and thus causes a slow drift of the bath temperature. It is customary to put a condenser in parallel with this contact but we have had much better results by arranging to use a small current of low voltage in the relay circuit and find that our mercury contact surface remains perfect for months at a time and makes and breaks contact with the platinum point at exactly the same temperature. A common 150-ohm telegraph relay was rewound with no. 36 wire so that the resistance of the magnet coils was about 1,000 ohms. The arm which controls the heating circuit was lengthened and adjusted to work very easily and only required .005 ampere in the relay circuit to operate it. This arrangement, without a condenser, has given excellent results.[1] We have had the best success with a very fine platinum point for the contact with the mercury surface. A narrow glass tube was drawn down to a point and a .05-mm. platinum wire sealed in so that only ½ a millimeter projected out beyond the glass point. A small copper lead wire was passed into

[1]The Weston Electrical Instrument Co. make a relay which operates on a low voltage circuit with a current of only .001 amp. and controls a current of 1 or 2 amps.

the open end of the tube and with a drop of mercury made contact with the platinum wire. This glass tube was fastened into a little 4-mm. brass tube which was threaded and screwed into the metal cap *c*, Fig. 5, and so a very fine adjustment of the bath temperature was possible; rough adjustments were made by adding or removing mercury from the tube *b*.

With the arrangements we have, the heating current is made or broken by a change in the bath liquid of only one or two thousandths of a degree. For constancy, and to avoid oscillations of the bath temperature about the desired point, it was necessary to have the bath well insulated and this was accomplished with the 6 cm. of excelsior packing between the tank and containing box and the plate glass cover. Also it was especially important to have a small amount of heat liberated in the unit of time, and well distributed in the bath liquid. Our heating coils consist of .25 mm. nickel wire wound on 10-mm. glass tubes. These tubes were 50 cm. long and the wire wound on with the aid of a lathe and a little guide which gave a pitch of about one millimeter to the windings. These tubes were shellacked to hold the wire in place and then four of them were fused to other tubes forming the arrangement indicated by Fig. 6. The windings were continued by hand on the new tubes and the ends soldered together so as to make one continuous coil with two free ends soldered to platinum wires which passed through the glass at the joints of the upright tubes. Contact was made with the platinum wires inside the upright tubes. The coils were now thoroughly shellacked. This heater was fastened to the lower part of the frame so that it was held in about the center of the bath liquid. The shellack insulated the wire from the bath liquid and has not been affected in the slightest degree by the kerosene while the heat generated in this coil is at once taken up by the bath liquid. The resistance of our coil is about 120 ohms and is connected with the 110-volt a.-c. lighting circuit, but we also use an external resistance as .4 ampere is sufficient to

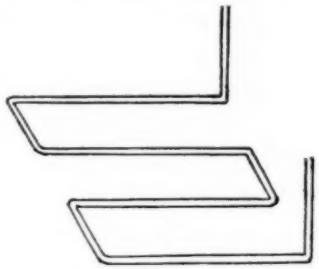

Fig. 6.

control the bath even with very considerable variations of the room temperature. The brass frame which carries the regulator and heater is covered with coarse wire netting so that the glass parts underneath the frame are protected, while the cells, in racks, rest on this netting.

It is not only necessary to have a constant temperature for the standard cells, but it is necessary to know what that temperature is. We use a set of 1/50° calibrated thermometers and they are checked at the ice point and the transition point of Glauber's salt (32.383). Some glass tubes pass horizontally through the side of the tank and box at various distances above the bath liquid and through these tubes the thermometers are read with a telescope. We have also checked our 25.00° temperature with a special 50 ohm resistance thermometer using 0°, 32.383 and 100° as the fixed points. Our known and reliable E.M.F. with a potentiometer, standard resistances, and standard batteries make it possible to determine the resistance of the coil of a resistance thermometer with ease and accuracy.

PRINCETON UNIVERSITY,
October, 1910.

A STUDY OF THE JOULE AND WIEDEMANN MAGNETOSTRICTIVE EFFECTS IN STEEL TUBES.

By S. R. Williams.

IN 1842 Joule[1] discovered that an iron rod changed its length when subjected to a magnetic field whose direction was parallel to the axis of the rod. This variation in length with change in magnetic field strength was found to be an increment up to a certain value of the magnetic field beyond which the rod appeared to contract. Later investigators[2] showed that if sufficiently strong magnetic fields were used the bar actually became shorter than when in its virgin state.

Some years before this interesting discovery of Joule, Wiedemann[3] found that "if a vertical wire is magnetized with its south end uppermost, and if a current is then passed downwards through the wire, the lower end of the wire, if free, twists in the direction of the hands of a watch as seen from above, or in other words, the wire becomes twisted like a right-handed screw if the relation between the longitudinal current and the magnetizing current is right-handed.

"In this case the magnetization due to the action of the current on the previously existing magnetization is in the direction of a right-handed screw around the wire. Hence the twisting would indicate that when the iron is magnetized it expands in the direction of magnetization and contracts in directions at right angles to the magnetization. This agrees with Joule's results." Thus Maxwell[4] explains the phenomenon in his second edition of Electricity and Magnetism and I have quoted directly because in the first edition it is stated oppositely and some of our recent[5] writers have persisted

[1] Joule, Phil. Mag. (37), vol. 30, pp. 76–225, 1847.

[2] Bidwell, Proc. Roy. Soc., 38, p. 265, 1885; 40, p. 109, 1886. More, Phil. Mag., 40, p. 345, 1895. Nagaoka, Rapports, Congress Internat. de Physique de 1900.

[3] Wiedemann, Elektricität, Bd. 3.

[4] Maxwell, Electricity and Magnetism, 2d ed., vol. 2, p. 87.

[5] Auerbach, Winkelmann's Handbuch der Physik, Elek. u. Mag., 2.

in giving the direction of rotation as Wiedemann[1] gave it originally, which is also incorrect.

Nagaoka and Honda[2] in their exhaustive studies on magnetostrictive effects in iron, nickel and cobalt, have indicated the direction of rotation even more correctly when they say: "The direction of twist in iron, so long as the longitudinal magnetizing field is not strong, is such that if the current is passed down the wire from the fixed to the free end and the wire is magnetized with north pole downwards the free end, as seen from above, twists in the direction of the hands of a watch." That is to say, if one increases the longitudinal magnetizing field from zero up to about three hundred units, c.g.s., the *initial* twist of the rod will be as Nagaoka and Honda have indicated, *viz.*, clockwise; but when the field strength reaches a value between 15 and 30 units a maximum twist in this direction occurs beyond which the twist appears to take place counter clockwise, *i. e.*, in the opposite direction and in a sufficiently strong longitudinal field the twist actually carries the scale reading to the opposite side of the zero point from that on which the maximum twist occurred. It is here perhaps that much of the confusion occurs. For instance, if a strong longitudinal field be imposed suddenly upon the iron bar, instead of gradually increasing it from zero up to that value, the twist will appear to occur counter clockwise, when the directions of the two fields are in a right-handed relation. In throwing on a strong longitudinal field it must necessarily build up from zero but occurring in such a short time the rod, due to inertia effects, does not twist to its maximum value but takes a mean position and then twists from that position to the one it holds for the maximum longitudinal field which gives it the appearance of twisting counter clockwise. I mention this question of direction of twist at some length because of the apparent confusion, when in reality there is none if we are careful to state what the magnitude of the longitudinal field is and whether the longitudinal field is suddenly or gradually built up. We must also keep in mind that the above holds for iron. In nickel the initial twist takes place in just the opposite direction and if we accept Maxwell's

[1] Wiedemann, Die Lehre vom Galvanismus, Bd. 2, p. 256, 1873.

[2] Nagaoka and Honda, Phil. Mag., 4, p. 61, 1902.

explanation that the Wiedemann effect is a special case of the Joule, then this is what we would expect, as a nickel rod shortens[1] in the Joule effect instead of lengthening as iron does for small magnetic fields.

This paper has for its object the study of these two effects (the Wiedemann and the Joule), in the same samples of steel tubing. So far as the author knows little or no comparative work has been done on the same specimens. In the Wiedemann effect the use of steel tubes enables one to thread an insulated copper wire through the tube so that the current for producing the circular field could be sent either through the wire or the tube itself as a conductor. Both cases were studied. With the current insulated from the tube, the circular field can be more readily calculated, for as Knott[2] points out we know nothing of the distribution of the circular magnetic field inside a solid iron conductor through which a current is flowing.

Description of Apparatus.

In Fig. 1 is shown a diagram of the apparatus as used in this work. C is the magnetizing coil, T, the steel tube with an insulated wire running through the center, X_1, the reversing switch for the current which flows through the steel tube, $Am.$, R_1, the ammeter and resistance in series with the circuit. For controlling the current in the solenoid, C, a reversing switch, X, was connected in series with a variable resistance, R, and ammeter, $Am.$, and a double pole double throw switch, D, whereby either alternating or direct current could be passed through C. The alternating current was used for demagnetizing the steel tubes before each set of readings. The switch, S, served as a short circuit for the ammeter when the alternating current was used. M is a tube containing mercury to serve as a connection for the lower free end of the steel tube and the wire running through them. With this arrangement of circuits the direction of the longitudinal and circular fields could be varied at will and any desired combination used in the Wiedemann effect.

The ammeters were calibrated from time to time by means of

[1] Barrett, Nature, 26, p. 585.

[2] Knott, Trans. Roy. Soc. Edinb., XXXII. (1), p. 193, 1883; XXXV, (2), p. 377, 1899; XXXVI. (2), p. 485, 1891.

a Leeds and Northrup potentiometer. The variable resistance, R, was a water resistance and was most satisfactory where one desired to vary the magnetic field continuously without any steps.

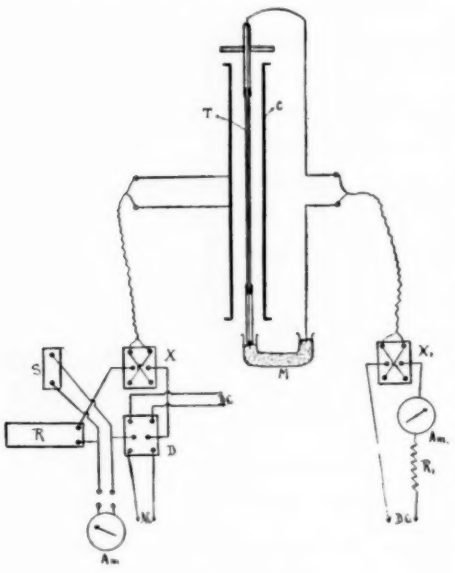

Fig. 1.

I am indebted to the Phœnix Physical Laboratory of Columbia University for the use of the solenoid, C, and wish hereby to express my appreciation of their kindness in loaning it. The coil is 100 cm. in length and wound in eight layers with 6,025 turns in all. It is wound on a very thick-walled brass tube with a slit running the full length of it. This thick-walled tube insured greater rigidity and also cut down heat changes in the space where the tubes were suspended.

I am particularly indebted to the Ellwood Ivins' Tube Works[1] for the excellent specimens of seamless steel tubing which they furnished me. There were four different sizes all of which were, according to manufacturers' notation, called "low carbon steel." For convenience I have designated these specimens, tubes A, B, C and D respectively. Their dimensions are as follows:

TABLE I.

Tube A.	Outer diameter,	.1600 cm.	Tube C.	Outer diameter,	.2447 cm.
	Inner diameter,	.0794		Inner diameter,	.0970
	Length,	79.8		Length,	79.7
Tube B.	Outer diameter,	.2386	Tube D.	Outer diameter,	.2088
	Inner diameter,	.1538		Inner diameter,	.1085
	Length,	80.2		Length,	80.0

The ends of the tubes, as the table indicates, were ten centimeters inside of the ends of the solenoid which gave fairly uniform fields.

[1] Address: Oak Lane Station, Philadelphia, Pa., U. S. A.

Heavy brass tubing was brazed to the ends of the steel tubes for supporting them and making connections. The tubes were suspended vertically in the solenoid and carefully placed along the axis of the same. The support for the tube was independent of the solenoid. A tripod support which rested directly upon the upper end of the solenoid was first tried but the results were very discordant until the supports for the tubes and solenoid were made independent. The whole system of supports was rigidly attached to a thick stone wall of the laboratory.

For a study of the Joule effect the mercury tube, *M*, Fig. 1, was removed and in its place was used a system of levers, shown in Fig. 2, for measuring the changes in length of the tubes. *LL'* is a light lever of which *L* is the fixed end. *T* is a continuation of the tube which supports the lever at *B* in a stirrup while *S* is the solenoid. At *L'* a silk cord drops downward and coils several times around a small roller to which is attached a mirror, *M*, for reading the deflection by means of a telescope and scale. A small weight, *W*, keeps the silk cord stretched. The multiplying power of the levers was determined by attaching the table of the dividing engine to the point, *B*, and obtaining the deflection of the mirror when *B* was displaced through a measured distance. The multiplying power was found to be 11,935. This system of levers worked very satisfactorily and would return to its old zero point after the demagnetization of the tubes. Attention is called to the roller on which the mirror was mounted. It had "agate" bearings which may be made in any laboratory. A piece of capillary tubing was broken off squarely and heated in a Bunsen flame until the end was thoroughly fused and the capillary opening commenced to close. At this point it is taken from the flame and the other end treated in the same way. These fused ends of the capillary tubes make excellent bearings

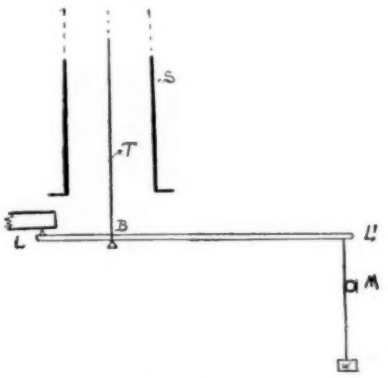

Fig. 2.

on steel needle points and have very small friction. The capillary tube may be used as a roller itself or short pieces of the tubing may be set into the ends of larger metallic cylindrical rods. In this work the glass tips were set in the end of a brass rod which was turned down to a small diameter where the cord passed around it. Fig. 3 shows more distinctly how it worked when supported by steel points.

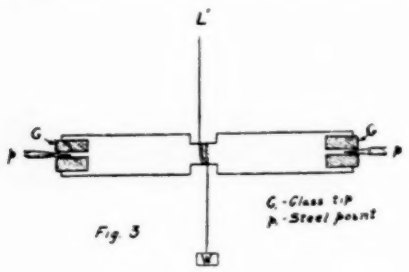

Fig. 3.

METHODS OF OBSERVATION.

In all of the specimens the following order of measuring the various tabulated results was used. First, the Wiedemann effect, second, the Joule effect, third, the permeability, fourth, the moments of torsion.

The values for the Wiedemann effect were obtained by keeping the current, producing the circular field, constant and by varying continuously the longitudinal field from zero up to the maximum value obtainable with the storage battery and coil which we have; this was about 275 c.g.s. units. The values thus obtained are the mean of four sets of readings taken in the following way:

TABLE II.

Vertical current flowing down.
Longitudinal field down.

Vertical current flowing up.
Longitudinal field down.

Vertical current flowing up.
Longitudinal field up.

Vertical current flowing down.
Longitudinal field up.

A reference to Fig. 1 will show that these various combinations were readily obtained by the connections and switches employed. The mean values thus obtained for the Wiedemann effect eliminated

the action of the earth's magnetic field and also the dissymmetry of field in setting up the apparatus. Before taking each set of readings the tubes were demagnetized by passing a decreasing alternating current through the solenoid. At least the scale readings returned to zero each time after such a treatment. It is evident that heating the tubes each time for demagnetization was out of the question.

Several values for the vertical current were taken for each tube and these values were repeated when the vertical current flowed along the tube itself as a conductor and not in the wire inside of the tube.

It was found that the direction of the initial twist was always the same whether the circular field was applied first or the longitudinal, but that the twist was smaller if the longitudinal was applied first. Further the two fields were made to vary simultaneously from zero upward by connecting the solenoid and vertical wire in series; the initial twist still maintained the same relation to the two directions of the circular and longitudinal fields. The following table illustrates the relation of the circular and longitudinal fields and the direction of initial twist as viewed from upper end of tube.

TABLE III.

Circular Field.	Longitudinal Field.	Direction of Twist.
Current down.	Down.	Clockwise.
Current up.	Down.	Counter-clockwise.
Current up.	Up.	Clockwise.
Current down.	Up.	Counter-clockwise.

The direction of the field in the solenoid was determined by a compass needle and the direction of the current in the tube by applying the terminals of a milli-voltmeter to the terminals of the tube and also by the usual test of the compass needle in the neighborhood of a conductor.

The values for the Joule effect are the means of two sets of readings, first, when the longitudinal field was up; second, when the longitudinal field was down, demagnetization occurring between each set of readings. The field was varied continuously.

The permeability of the tubes was determined by the ballistic method and the results are the mean of several sets of readings.

The moments of torsion were determined in a torsion lathe. Two mirrors were attached to the tubes and the twist determined

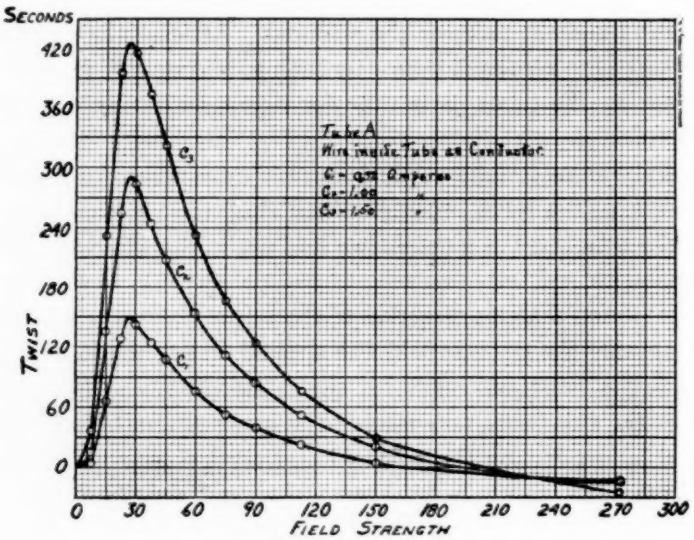

Fig. 4.

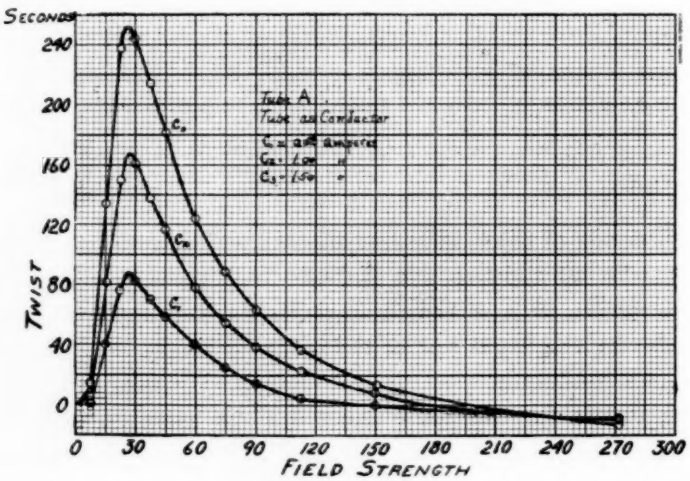

Fig. 5.

from the deflections of the mirrors as seen with two telescopes and scales. This eliminated any slipping at the chucks.

TABLE IV.

Tube A.

Longitudinal Field.	Total Twist of Tube in Seconds of Arc.						Total Lengthening in Centimeters.	Permeability.	Moment of Torsion.
	Vertical Current in Wire. Amperes.			Vertical Current in Tube. Amperes.					
	0.52	1.00	1.50	0.52	1.00	1.50			
	Twist.			Twist.					
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
7.57	4.7	15.7	36.1	0.8	5.8	14.5	0.0	233.08	
15.14	66.3	135.5	231.8	40.0	81.2	134.2	7.46×10^{-7}	389.05	
22.71	128.4	254.1	395.6	76.1	149.2	237.5	———	443.98	
30.28	143.4	282.9	415.3	82.4	160.9	244.2	2.03×10^{-5}	409.09	615 gm. cm.
37.85	124.5	243.5	373.2	70.6	138.1	213.9	———	355.61	
45.42	108.0	209.0	322.1	59.6	116.5	180.2	2.61×10^{-5}	316.78	
60.57	76.6	154.4	232.5	40.0	78.5	125.6	2.82×10^{-5}	255.20	
75.71	53.8	111.9	167.7	25.5	55.7	89.1	2.82×10^{-5}	203.80	
90.85	39.2	84.4	123.7	15.7	39.2	63.6	2.82×10^{-5}	182.62	
106.00	——	——	——	——	——	——	———	161.74	
113.56	23.5	53.0	77.3	4.7	23.5	37.2	———	——	
151.42	4.7	21.5	29.4	0.0	8.6	13.7	2.73×10^{-5}	115.02	
272.56	−13.7	−13.7	−25.5	−8.6	−9.8	−13.7	2.06×10^{-5}	85.94	

TABLE V.

Tube B.

Longitudinal Field.	Total Twist of Tube in Seconds of Arc.						Total Lengthening in Centimeters.	Permeability.	Moment of Torsion.
	Vertical Current in Wire. Amperes.			Vertical Current in Tube. Amperes.					
	1.00	1.50	2.00	1.00	1.50	2.00			
	Twist.			Twist.					
0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.00	
7.57	53.0	57.1	113.3	4.8	14.8	18.9	0.00	706.32	
15.14	167.6	234.0	323.7	45.0	63.1	73.1	4.58×10^{-6}	739.40	
22.71	181.7	266.1	357.9	56.2	75.1	90.4	———	602.90	2,704
30.28	170.8	246.0	333.6	56.2	73.1	91.2	1.36×10^{-5}	470.61	gm. cm.
37.85	150.7	217.9	293.5	51.0	69.5	82.4	———	390.64	
45.42	130.6	189.0	256.1	46.6	64.4	75.1	1.61×10^{-5}	329.31	
60.57	101.3	144.7	197.0	40.2	53.0	63.1	1.65×10^{-5}	267.38	
75.71	77.2	110.5	151.5	34.1	45.0	54.2	1.65×10^{-5}	219.87	
90.85	63.1	88.4	119.4	28.9	37.7	45.0	1.64×10^{-5}	190.41	
106.00	——	——	——	——	——	——	———	164.55	
113.56	44.2	63.1	84.4	18.9	30.9	34.9	———	——	
151.42	20.9	28.9	48.2	12.5	22.1	26.9	1.56×10^{-5}	120.66	
272.56	2.8	2.0	8.8	4.0	10.8	16.8	1.25×10^{-5}	82.18	

Tables IV., V., VI. and VII. give the various results for tubes *A*, *B*, *C* and *D* respectively, which are plotted in Figs. 4, 5, 6, 7, 8,

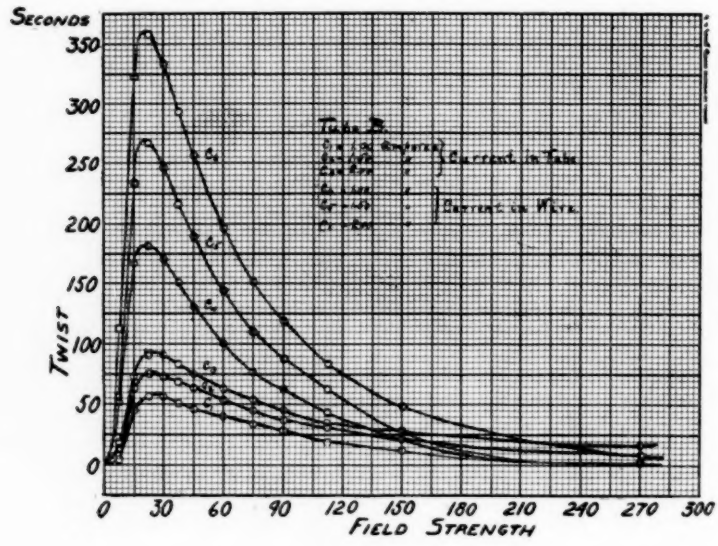

Fig. 6.

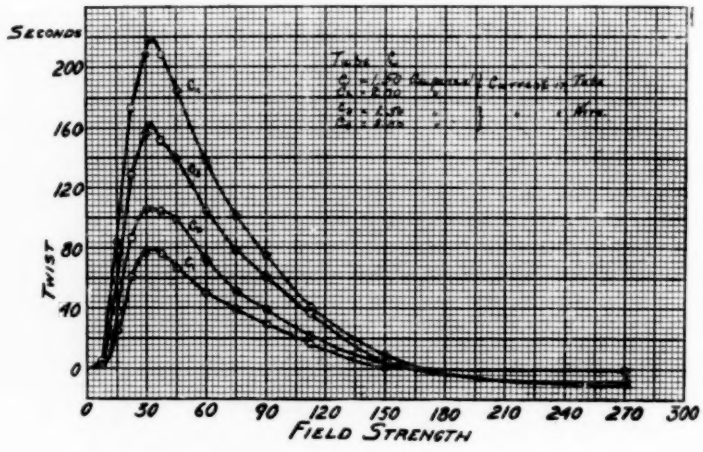

Fig. 7.

9, 10, 11 and 12. Fig. 4 shows the total twist for tube *A* when three different vertical currents were used and which flowed along the insulated wire inside of the tube. Fig. 5 represents the total twist

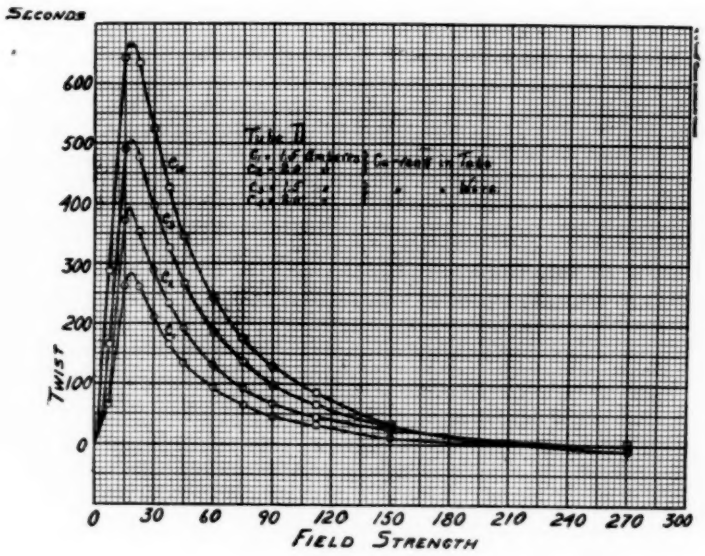

Fig. 8.

TABLE VI.

Tube C.

Longitudinal Field.	Total Twist of Tube in Seconds of Arc.				Total Lengthening in Centimeters.	Permeability.	Moment of Torsion.
	Vertical Current in Wire. Amperes.		Vertical Current in Tube. Amperes.				
	1.50	2.00	1.50	2.00			
	Twist.		Twist.				
0.00	0.0	0.0	0.0	0.0	0.00	0.00	
7.57	2.8	4.8	0.8	0.0	0.00	133.51	
15.14	58.8	84.1	24.8	38.0	0.00	381.69	
22.71	128.2	173.1	60.9	86.9	———	440.72	3,504
30.28	156.3	208.4	76.1	106.2	0.53×10^{-5}	404.11	gm. cm.
37.85	151.1	208.4	76.1	105.0	———	362.56	
45.42	139.0	185.2	66.9	99.0	1.28×10^{-5}	322.15	
60.57	105.0	138.2	50.1	72.9	1.59×10^{-5}	265.30	
75.71	78.1	100.2	38.8	51.0	1.59×10^{-5}	220.98	
90.85	60.0	74.9	29.6	38.8	1.59×10^{-5}	191.58	
106.00	——	——	——	——	———	168.37	
113.56	36.8	40.0	16.4	22.0	———	———	
151.42	4.0	8.0	2.8	6.0	1.55×10^{-5}	120.83	
272.56	−8.8	−10.8	0.0	−0.8	1.34×10^{-5}	72.27	

for *A* when the same vertical currents as in Fig. 4 were used, only they flowed in the tube itself. Fig. 6 shows the same thing for tube *B*, the three upper curves for the case when the current is flowing in the wire inside of the tube and the three lower when the tube was the conductor. In all of the tubes the maximum twist was greater when the current was insulated from the tube than when flowing in the tube itself. Similarly Figs 7 and 8 show the total twist for tubes *C* and *D* respectively with the two upper curves for current in wire inside of tube and the two lower curves for the vertical current in the tube. Fig. 9 represents the total twist for the four tubes under exactly the same conditions. In these curves the vertical current was insulated from the tubes. Fig. 10 indicates the Joule effect for the four tubes under the same conditions. Figs. 11 and 12 show their permeability.

TABLE VII.

Tube D.

Longitudinal Field.	Total Twist of Tube in Seconds of Arc.				Total Lengthening in Centimeters.	Permeability.	Moment of Torsion.
	Vertical Current in Wire. Amperes.		Vertical Current in Tube. Amperes.				
	1.50	2.00	1.50	2.00			
	Twist.		Twist.				
0.00	0.0	0.0	0.0	0.0	0.00	0.00	
7.57	167.9	288.0	69.2	75.3	0.00	668.74	
15.14	490.2	642.6	263.8	372.6	0.31×10^{-5}	774.99	
22.71	477.3	634.5	262.6	352.5	———	631.90	1,876
30.28	396.8	524.5	211.5	288.8	1.56×10^{-5}	506.04	gm. cm.
37.85	324.2	427.0	165.1	234.4	———	412.47	
45.42	266.6	347.2	135.7	192.1	1.69×10^{-5}	354.87	
60.57	186.1	241.7	92.6	130.9	1.68×10^{-5}	277.21	
75.71	136.9	175.2	65.2	91.4	1.65×10^{-5}	225.10	
90.85	99.5	129.7	47.1	69.2	1.63×10^{-5}	194.43	
106.00	——	——	——	——	———	167.49	
113.56	66.4	80.5	31.0	45.1	———	———	
151.42	30.2	34.2	10.8	26.1	1.51×10^{-5}	122.07	
272.56	−2.8	−8.8	−0.8	4.8	1.17×10^{-5}	81.94	

Discussion of Results.

In Figs. 9, 10, 11 and 12 are shown the lengthening and twist and permeability of the four tubes. It is interesting to note that

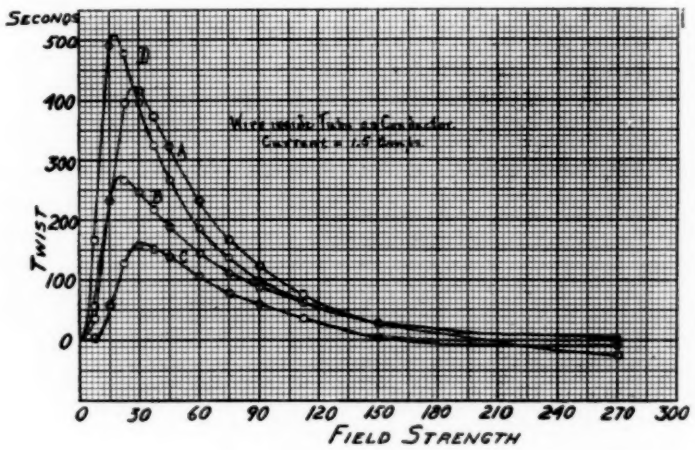

Fig. 9.

the curves for lengthening and twist of the tubes *A* and *C* have this characteristic in common with the permeability curves that their

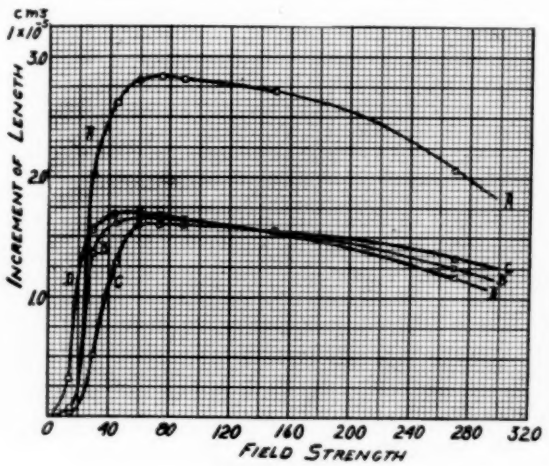

Fig. 10.

maxima occur at a higher value of the longitudinal field than do those of the tubes *B* and *D*. Apparently the permeability is the

only property of the tubes which gives a decided color to the character of the curves of twist and lengthening. From the curves

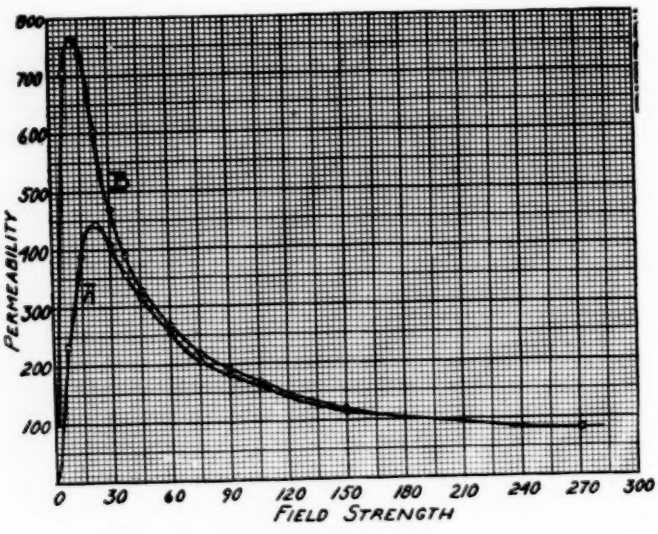

Fig. 11.

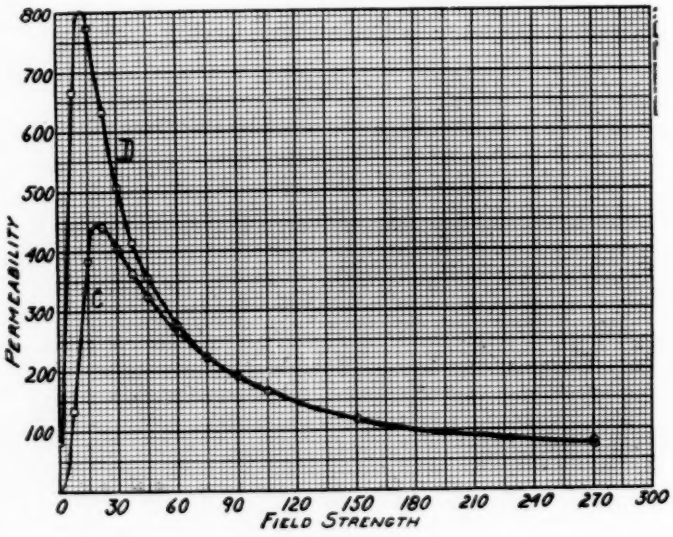

Fig. 12.

showing the Wiedemann and Joule effects it would also appear that the permeability influenced the twist more than the lengthening.

In no other way, it seems, can one account for the general results which I have tabulated in the following manner:

TABLE VIII.

Maximum twist,	$D > A > B > C.$
Maximum lengthening,	$A > D > B > C.$
Maximum permeability,	$D > B > A > C.$
Moment of torsion,	$A < D < B < C.$
Mean radius,	$A < D < C < B.$

From Maxwell's explanation one might be led to suppose that in the steel tube showing the maximum lengthening one would find a maximum twist. This does not hold for all of the tubes as shown by the curves for A and D. Their moments of torsion, mean radius and lengthening all indicate that A ought to twist more than D for a given torque, but such is not the case and although I repeated my results with numerous imposed conditions the relations still held. If the permeability is an important factor in the twist, we would expect D to twist more than A, since the maximum permeability of $D \succ B \succ A$. This reasoning however does not hold for tubes A and B when they are compared, but here either the moment of torsion or the mean radius at which the circular field is acting may be the important factor, as their difference in permeability is not as great as that of D and A. The results at least lead to the conclusion that there is no simple relation between the Joule and Wiedemann effects.

The comparative study of the twist in the Wiedemann effect when the vertical current is flowing in the wire and when flowing in the tube show that the maximum twist is from two to four times as great in the first case as in the second. I suppose this is to be ascribed to the fact that when the vertical current is flowing in the tube itself as a conductor there is not as large a circular magnetic field operative on the tube as when the same current is flowing in the wire, for there is no magnetic force in the interior of a cylindrical tube conveying a current.

As pointed out at the beginning, the use of tubes in the study of the Wiedemann effect offers some advantages over solid rods: first, one can determine the circular magnetic field when the current is confined to an insulated wire inside of the tube, secondly, for the

same vertical current the Wiedemann effect is increased when the current flows in the wire.

For the tube A, the moment of torsion equals 615 gram-centimeters. The maximum twist for a vertical current of 1.5 amperes is equal to 415.3 seconds. This means that a torque of about 1.24 gram-centimeters would have been necessary to have produced the same twist. If we take as our mean radius of A a value of .06 centimeters it would mean a force of 20 + grams weight acting at that arm's length to produce the effect. It is interesting to consider in connection with this that due to the circular magnetic field about the vertical current that the lower free end of the tube, since it is a magnetic pole in the Wiedemann effect, tends to rotate about the current with a torque equal to $2mI$ where m equals the pole strength and I is the current. The direction of this torque would be that of the initial twist in the Wiedemann effect. Computing this for the case where the permeability is a maximum in the tube A shows that this torque has a value of only a few dyne-centimeters and hence may be neglected.

If the equipment had been available I would have preferred to have carried out these experiments on a much larger scale, *viz.*, by using tubes several meters long and consequently of larger diameters and corresponding solenoid for producing the longitudinal fields. Limited thus I present the results obtained from the equipment I had at my disposal.

Mr. Dorsey[1] in a recent article on the Joule effect gives an excellent bibliography of the subject; most of the articles referred to were available in this work.

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OBERLIN, OHIO.
November 5, 1910.

[1] Dorsey, PHYS. REV., vol. 30, p. 698, 1910.

THE CAUSES OF ZERO DISPLACEMENT AND DEFLECTION HYSTERESIS IN MOVING-COIL GALVANOMETERS.[1]

By Anthony Zeleny.

§ 1. Zero Displacement after a Deflection.

IN a moving-coil galvanometer the displacement in the zero reading which takes place after a first deflection, notwithstanding the conclusion of A. Stansfield,[2] has generally been ascribed to a true set in the suspension fiber. The following investigation proves the view to be erroneous and that practically the whole effect is due to the action of magnetic impurities within the coil.

A galvanometer coil, G, was suspended within a wooden frame between the poles of a large electromagnet as shown in Fig. 1. If the zero displacement is due to a true set in the fiber its magnitude after reversed deflections of equal size would be independent of the intensity of the magnetic field, but if the effect is due to magnetic impurities within the coil the magnitude of the displacement would increase with the intensity of the field.

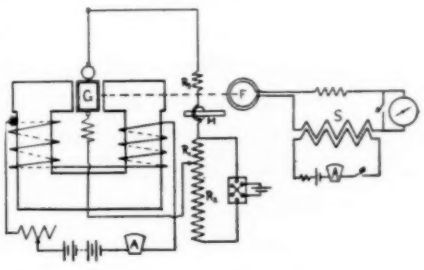


Fig. 1.

The deflecting current was varied so as to produce the same deflection with each altered intensity of the field. In all cases the coil was made to move aperiodically to its deflected position as well as to its zero or null reading, in stronger fields by placing the proper resistance in R_3 of the galvanometer circuit and in weaker ones by induced currents from an accessory coil and magnet M. The

[1] Preliminary report read before Section B, A.A.A.S., Baltimore Meeting, 1908–9; abstract in Science, March 19, 1909, p. 471.

[2] Phil. Mag., vol. 46, p. 67, 1898.

intensity of the field was measured by jerking the test coil *F* through the field by means of a spring and comparing the ballistic galvanometer throws with those obtained from a standard inductance coil[1] *S*. The test coil had the same area as the pole pieces and measured the intensity of the field in the immediate neighborhood of the poles. The galvanometer coil *G* was at some distance from the poles, and the intensity of that part of the field containing it was obtained by multiplying the values given by the test coil by the approximate, experimentally determined factor 0.768.

The galvanometer coil was taken from a Leeds and Northrup galvanometer (Type P, no. 10598), and was the one of several tested that contained the smallest amount of magnetic impurities (see § 5 and Table IV.). The coil is a hollow rectangle, 2.05 by 5.1 cm., and has a mass of 6.66 grams and a moment of inertia of 3.49 c.g.s. units. The upper suspensions employed were of phosphor-bronze strip and the lower suspension was an ordinary 1.5 mil phosphor-bronze strip spiral of 7.5 turns, each loop being 5 mm. in diameter. The torsional moment when employing a 3-mil upper suspension was 2.37 c.g.s. units and when employing a 1.5-mil suspension, 0.24 c.g.s. units. Unless otherwise stated the distance of the circular reading scale in all cases was 54.5 cm.

The observations for the zero displacement of the coil for any particular field intensity were taken at equal intervals of one minute (see § 4), after several successive deflections alternating in direction. The zero displacement after the first reversal was always considerably larger than that observed after the following ones which gave nearly equal though consistently diminishing values. The following observations give a fair illustration of the relative values of the displacement after successive reversals: 2.41, 2.08, 2.06, 2.05 cm. The displacement of the null reading in fields of different intensity, when using the 3-mil upper suspension and reversing the current from a deflection of 20 cm., is given in Table I., the displacement after the first reversal being discarded and the average of the following three taken.

[1] A. Zeleny, PHYS. REV., vol. 23, p. 411, 1906. E. B. Rosa, PHYS. REV., vol. 24, p. 241, 1907.

TABLE I.

Intensity of Field.	Zero Displacement after Reversal.
6	.015 cm.
86	.025
169	.05
220	.09
384	.24
614	.54
1,046	1.24
1,400	1.73
1,750	2.06
2,009	2.25
2,177	2.33

The data of Table I. are plotted in the curve *A* of Fig. 2. It is observed that in fields whose intensity is less than 100 the displacement curve is nearly horizontal and the point where it cuts the axis of ordinates represents the amount of displacement due to a true set in the suspension fiber. This however is so small that under ordinary conditions of use it would rarely be observable. Under the conditions given, the true set in the fiber is represented by a displacement of 0.015 cm., while the zero displacement in a field whose intensity is 400, such as is common in ordinary galvanometers, is 0.26 cm. With this coil and suspension, less than 6 per cent. of the observed zero displacement in a galvanometer is due to a true set in the suspension fiber and more than 94 per cent. to the magnetic hysteresis within the coil.

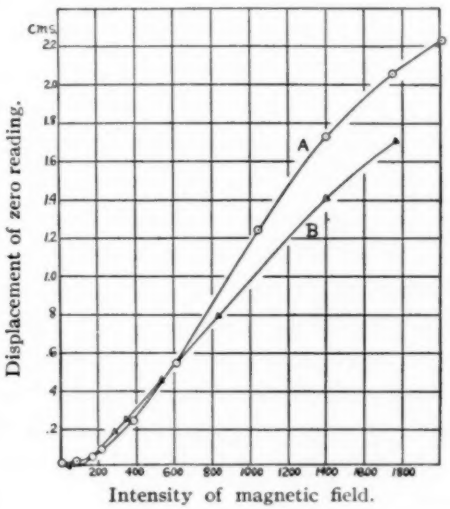


Fig. 2.

With a 1.5-mil phosphor-bronze strip suspension the zero displacement, under the same conditions as above, gave in the weakest field a displacement not exceeding 0.005 cm., or one third that

obtained with the 3-mil strip. It is seen, therefore, that the larger zero displacement observed in galvanometers with the thinner suspensions is due entirely to the magnetic properties of the coil and not at all to a greater set in the fiber. The magnitude of the torsional moment of the suspension practically alone determines the amount of the zero displacement for the same alteration in the magnetic moment produced by the reversal.

The results of a set of observations taken with a 1.5-mil phosphor-bronze strip suspension are given in Table II. and are plotted in curve *B* of Fig. 2. In this case the reversals were taken from a deflection of 15 cm. in place of 20; and the distance of the scale was 125 cm. in place of 54.5.

TABLE II.

Intensity of Field.	Zero Displacement after Reversal.
41	.000 cm.
291	.175
346	.24
538	.45
837	.79
1,402	1.40
1,766	1.70

The curves *A* and *B* are not coincident or parallel to each other because the deflections and the distances were not the same in the two cases and, in addition, the position of the zero point changed with the intensity of the field and by different amounts in the two cases, so that in fields of the same intensity the lines of force did not have the same relative direction with respect to the coil.

In the case given in Table II. the zero displacement in the weakest field was too small to be observable, and, therefore, no appreciable set existed in the suspension fiber in any of the displacements observed.

§ 2. CHANGE IN THE ZERO READINGS WITH THE INTENSITY OF THE MAGNETIC FIELD.

The change in the zero reading with the intensity of the magnetic field is shown for the two cases of § 1, in Fig. 3. The change is approximately proportional to the intensity of the field, and for

the 1.5-mil suspension is approximately five times that for the 3-mil suspension. The changes although observed with the scale at a distance of 125 cm. are plotted in the curve *D* from values reduced to what they would have been at the same distance as that of the similar observations with the 3-mil suspension. In each curve the change of zero is reckoned from its observed value in the weakest field.

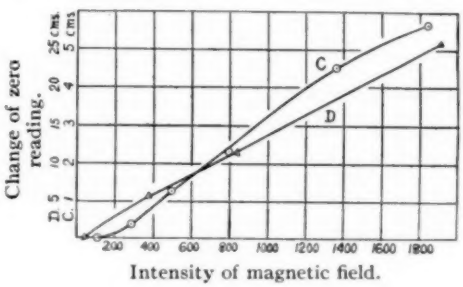


Fig. 3. *C*, 3-mil suspension; *D*, 1.5-mil suspension.

The plane of the coil was set perpendicular to the faces of the poles of the electromagnet in the weakest field employed, and it turned visibly from this position as the field increased, suggesting that the magnetic impurities were not uniformly distributed through the body of the coil; but it was shown experimentally that at least a considerable part of the change was due to an alteration in the direction of the magnetic field. The position of the coil from which the same current gave equal deflections on the two sides of the zero, shifted with the change of intensity of the field.

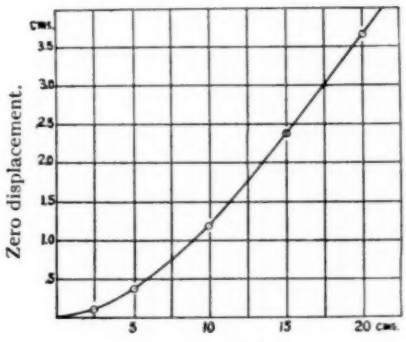


Fig 4.

§ 3. Relation of the Zero Displacement to the Magnitude of the Reversed Deflection.

The relation of the zero displacement to the magnitude of the reversed deflection is shown in Fig. 4. The 1.5-mil upper suspension was employed, and the intensity of the magnetic field was 1,244 c.g.s. units.

§ 4. The Change with Time of the Zero Reading after a Reversed Deflection.

The zero reading obtained after a reversed deflection changes slowly with time. This is shown in Table III. for a 1.5-mil suspension fiber and a field of 1,244 c.g.s. units, when the zero displacement at the end of one minute was 3.34 cm.

TABLE III.

Time.	Change in Zero Displacement.
10 sec.	(0.00) assumed.[1]
30 "	0.03 cm.
1 min.	0.05
5 "	0.08
60 "	0.09

§ 5. Magnitude of the Magnetic Moment of Some Galvanometer Coils.

Several coils were suspended in succession in a magnetic field of 307 c.g.s. units. The upper suspension was a 1.5-mil phosphor-bronze strip 12.3 cm. in length, which with the lower spiral had a torsional moment of 0.171 c.g.s. units. The period of vibration in each case was determined first without and then within the field. Letting t_1 and t_2 = the periods of vibration, T = the torsional moment, and M = the magnetic moment due to the action of the field upon the impurities less the smaller moment due to the diamagnetic property of the copper, then

$$t_1 = 2\pi\sqrt{\frac{I_0}{T}} \quad \text{and} \quad t_2 = 2\pi\sqrt{\frac{I_0}{T+M}}; \quad \text{and} \quad \frac{M}{T} = \frac{t_1^2 - t_2^2}{t_2^2}.$$

Table IV. gives the results obtained for the several similar galvanometer coils. The magnetic moment in these cases had nearly the same magnitude as the torsional moment.

TABLE IV.

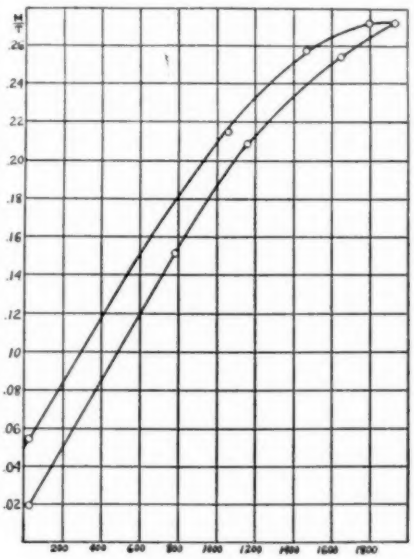
Coil Number.	$\frac{M}{T}$	M
6,788	0.87	0.144
9,127	1.31	0.217
10,597	0.82	0.136
10,598	0.78	0.129
	0.95	0.157

[1]It required 10 seconds to bring the coil to rest after a deflection.

The 3-mil suspension with the lower spiral was found to have a torsional moment $T' = 2.37$ c.g.s. units, which with the above average value of M makes $M/T' = 0.066$.

§ 6. Deflection Hysteresis.

In a former paper[1] on Precision Measurements with the Moving-coil Galvanometer the writer observed that deflections of different magnitude were obtained with the same current, depending on whether they reached their value from a larger or a smaller one. This was explained as due to a change in the intensity or direction of magnetization of the impurities within the coil. Although the deflecting moment remained the same the restoring moment was changed by an alteration in the magnetization of the coil. In order to account for the observed changes, it was necessary to assume that the coercive power of the magnetic impurities was very large. To prove, experimentally, that this is so, the values of M/T were now obtained in various fields with the intensity increasing to a maximum and then decreasing. The hysteresis curve obtained, when using a 3-mil suspension fiber, is plotted in Fig. 5.



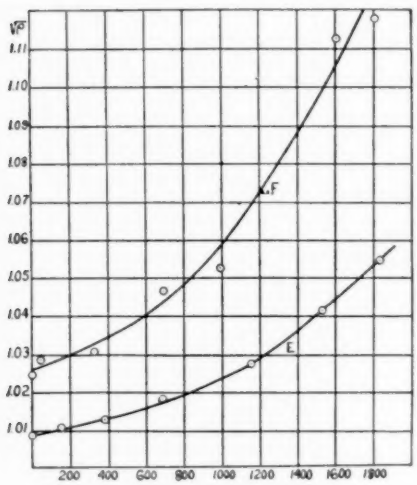
Intensity of magnetic field.

Fig. 5.

The coercive power is found to be sufficient to account for the observed deflection hysteresis. It will be seen, also, that the upper bends in the curve begin, as should be the case, in a field of the same intensity as the upper bends of the zero displacement curves in Fig. 2. These bends occur in a field whose intensity is more than 200 times that which produces the same effect in the case of iron.

[1] Phys. Rev., vol. 23, p. 401, 1906.

The shape of the curve is slightly affected by the change in the zero position of the coil (see § 2). The damping factor changed with the intensity of the field as given in § 7, but not enough to affect the period of vibration sufficiently to make corrections necessary for the purpose under consideration. The stray magnetism was found to have no appreciable effect upon the torsional moment of the suspension fiber.



Intensity of magnetic field.

Fig. 6.

§ 7. Change in the Damping Factor with the Intensity of the Field.

It is of some interest to note the experimental values of the damping factor for the same coil in fields of different intensity. Table V. gives the observed values of $\sqrt{\rho}$ on open circuit and of M/T, for several field intensities, when using a 3-mil phosphor-bronze suspension. The resistance of the galvanometer coil with the suspension was 116 ohms.

TABLE V.

Field Intensity.	$\frac{M}{T}$	$\sqrt{\rho}$
0	(0.000)	1.0088±5
154	.038	1.0108
373	.075	1.0127
685	.133	1.0180
1156	.204	1.0272
1530	.250	1.0411
1832	—	1.0541

These values of $\sqrt{\rho}$ are plotted in the curve E of Fig. 6, and similar values for the 1.5-mil suspension, in the curve F of the same figure. The irregularity of the observed points is due mainly to the difference in the magnitude of the arcs of vibration employed, which varied from 6 to 20 cm. on a scale 122 cm. distant.

It is seen from the curves that in field intensities of 400, common to galvanometers, when the coil is on open circuit, the damping due to the induced currents is less than 50 per cent. of that due to the resistance of the air and the internal friction of the fiber.

§ 8. Change of Sensibility with Alteration of Intensity of the Magnetic Field.

The sensibility of the galvanometer formed by the electromagnet and the coil was found, as expected, to increase at a smaller rate than the intensity of the field. The observed values, when using a 3-mil suspension and having the reading scale at a distance of 60.5 cm., are given in Table VI. The suspension, however, was not the identical one used in the previous observations.

Table VI.

Intensity of Field.	Current per 1 mm. Deflection.	Current per 1 mm. Deflection per Field Intensity of 100.
445	1.53×10^{-8}	6.8×10^{-8}
932	$.84\times10^{-8}$	7.8×10^{-8}
1315	$.70\times10^{-8}$	9.2×10^{-8}

§ 9. Summary.

The zero displacement observed after a deflection in a moving-coil galvanometer is shown, for phosphor-bronze suspensions, to be due almost entirely to the change in the intensity and direction of magnetization of the magnetic impurities within the coil. The amount due to a true set in the fiber is negligible under ordinary conditions of use, and in the cases investigated the largest true set observed was less than 6 per cent. of the whole zero displacement.

The hysteresis curve of the magnetic impurities within the coil shows a very large coercive power sufficient to account for the observed deflection hysteresis in moving-coil galvanometers. The upper bend of the curve is at a field intensity of about 1,600, which is great compared to the corresponding intensity for iron or steel.

During the progress of the experiments several other properties of moving-coil galvanometers were investigated:

1. The relation of the zero displacement to the magnitude of the reversed deflection, and the change of the zero reading with time, were determined.

2. The moment due to the magnetism in the galvanometer coils was found to differ considerably even in instruments of the same type and from the same manufacturer. In the similar coils tested, the magnetic moment, less the diamagnetic moment, in the same field varied from 0.129 to 0.217 c.g.s. units.

3. The values of the damping factor for the same coil in fields of different intensities were determined. More than two thirds of the damping on open circuit was found, for ordinary galvanometers, to be due to the resistance of the air and the internal friction of the fiber, and less than one third to the induced currents.

4. The change of sensibility with the intensity of the field was determined with a coil of known magnetic properties and a suspension of approximately known torsional moment.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
November 16, 1910.

REMARKS ON A PAPER BY J. S. STOKES ON "SOME CURIOUS PHENOMENA OBSERVED IN CONNECTION WITH MELDE'S EXPERIMENT."[1]

By C. V. Raman.

AT the conclusion of the paper referred to, the writer remarks: "As far as the demonstrator has been able to ascertain, these phenomena have not been hitherto observed or described by any one." I may therefore be permitted to observe that the phenomena described in the paper (with the exception of the effect on the rotation of the pulley of waxing the thread) were observed by me about five years ago, when working in collaboration with Mr. V. Apparao at the Presidency College, Madras, and were subsequently shown by us to a large number of others. As however, other phenomena of interest were then observed which seemed to be of greater importance and which could be explained less readily, I did not seek an opportunity of publishing my observations on the subject.[2]

I may also state that the phenomena of the rotation of the pulley was independently observed about two years ago by Mr. A. W. Porter, who published a note on the subject in Knowledge and Scientific News at that time.

My observations furnish a clue for the explanation of an effect which Mr. J. S. Stokes says he is unable to account for, *i. e.*, the increased speed of rotation of the pulley when the overhanging vertical portion of the string was vibrating and the horizontal part seemed not to be doing so. In a future paper I shall endeavor to show that with the large amplitudes of oscillation maintained in practice, Lord Rayleigh's theory of the maintenance of oscilla-

[1] Phys. Rev., May, 1910, p. 659.

[2] For the outcome of some of the other observations mentioned, the following publications in Nature may be referred to: On the Small Motion at the Nodes of a Vibrating String, Nature, November 4, 1909; The Maintenance of Forced Oscillations of a New Type, Nature, December 9, 1909; also an addendum to the latter in Nature, February 10, 1910.

tions by forces of double frequency[1] has to be seriously modified to enable the experimentally observed phase-relations to be accounted for. It comes out that we are not dealing with one, but with two variations of tension: not merely with the variation of tension *imposed* by the motion of the prong, but also with the periodic part of the variation of tension associated with an oscillation of large amplitude. In actual practice, these two variations approach equality in amplitude and opposition in phase, with the result that when the string is maintained in vigorous oscillation, the net periodic variation of tension is very small indeed compared with what we should expect from the range of excursion of the end of the string which is attached to the prong; the torque exerted upon the pulley and the speed of rotation, if any, of the latter are quite small. But when the string between the fork and the pulley has no transverse oscillation, the full effect of the oscillation of the prong is felt in varying the tension of the string periodically, and the torque on the pulley being greater, the speed of rotation of the latter becomes very considerable. This is exactly what is observed.

[1] Phil. Mag., 1887, and Theory of Sound, 2d ed., Vol. I., pp. 82–84.

THE SMALL MOTION AT THE NODES OF A VIBRATING STRING.[1]

By C. V. Raman.

IT is generally recognized that the nodes of a string which is maintained permanently in oscillation cannot be points of absolute rest, as the energy requisite for the maintenance of the vibrations is transmitted through these points. I have not however seen anywhere a discussion or experimental demonstration of some peculiar properties of this small motion. I shall therefore endeavor to give an account of some experiments and observations of mine relating to this subject.

2. Some rather striking effects are observed when the small motion at a node is viewed stroboscopically, *i. e.*, under periodic illumination. For this purpose, the frequency of intermittence of the light should be nearly twice that of the oscillation of the string. A tuning-fork maintains the string in oscillation in any convenient number of loops, by imposing a transverse obligatory motion at one point of it. Another tuning-fork, which has nearly twice the frequency of the other, forms the interrupter of a Ruhmkorff's coil, the spark from which furnishes the periodic source of illumination. Both forks are electrically self-maintaining. The string is seen in *two* slowly-moving positions, which represent opposite phases of the actual motion. If the nodes were points of absolute rest, then the two positions seen under the periodic illumination would intersect at fixed points. On account, however, of the small transverse motion at the nodes, the points of intersection or "fictitious nodes" are seen to execute a motion of *large* amplitude parallel to the string—the range of the motion being equal to the whole length of a loop. This motion, best seen under a magnifying glass, is represented in Fig. 1, in which nine successive stages at equal intervals of a complete cycle are shown.

[1] A preliminary note on this subject was published in Nature, November 4, 1909, as a letter to the Editor.

3. The "fictitious" node is in the first stage at the center of the field. It then moves to one side of the field, first slowly, then more rapidly; at the fourth stage, it is off the field; at the fifth, the two positions of the string are, at the center of field (*i. e.*, at the position of the node in the actual oscillation), sensibly parallel to each other. It then reappears on the other side of the field, moves in rapidly, then more slowly; at the ninth stage, it is back again at the center of the field.

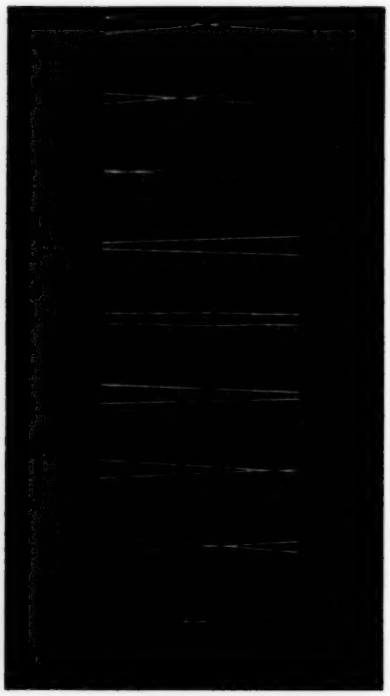


Fig. 1.

3. A motion of the type shown in Fig. 1 can be represented mathematically by the expression $A \sin \alpha x \cos pt + B \cos \alpha x \sin pt$, where x is the distance from the center of the field, t is the time, the other quantities being constants or nearly so. The two terms differ in phase by quarter of an oscillation. The significance of this is that the small motion at the node and the large motion elsewhere differ in phase by quarter of an oscillation. If the expression for the displacement were of the type $A \sin \alpha x \cos pt + B \cos \alpha x \sin (pt + E)$, the two terms differing in phase by more or less than $\pi/2$, the motion would not be of the type shown in Fig. 1. It would be unsymmetrical, the velocity of the point of intersection when at a given distance from the center of the field on one side and approaching it, being much greater than its velocity when at the same distance on the other side and receding from the center.

5. Another experiment, which was first performed by me in collaboration with Mr. V. Apparao, of the Presidency College, Madras, was found later to furnish a second method of determining the phase of the small motion at the nodes. The principle of this

method was to compound the oscillation at every point on the string with another perpendicular to it of half the frequency and to observe the compound oscillation at the nodes and elsewhere. A string can be maintained permanently in a compound oscillation of this character by attaching one end of it to the prong of an electrically-maintained tuning-fork, so that it lies in a plane perpendicular to the prongs, but in a direction inclined to their line of vibration.[1]

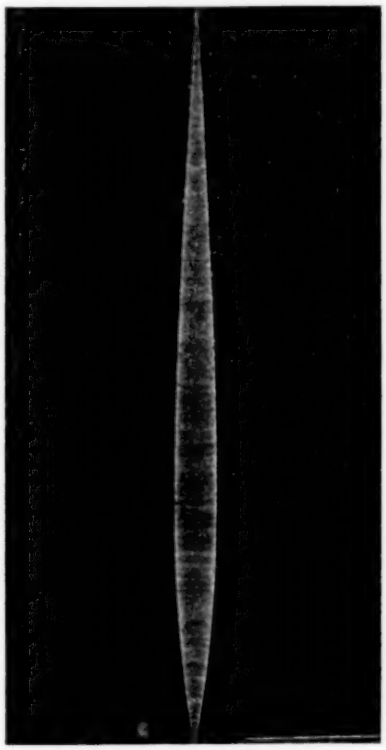


Fig. 2.

6. A beautiful and interesting type of stationary oscillation is maintained when the tension is somewhat greater than that necessary for the most vigorous maintenance. The curves described by points on the string are then parabolic arcs.[2] As the frequency of oscillation in one plane is half that in a perpendicular plane, there are *two* vibration-loops in the latter for every *one* in the former. The consequence of this is that the parabolic arcs which form the paths of points on the string have their curvatures in opposite directions in alternate halves of a big loop, *i. e.*, in alternate small loops. The surface generated by the moving string is one of great delicacy and purity, and an adequate idea of it can only be had on actually performing the experiment. The photograph herewith

[1] Under these circumstances, the motion of the prong may be resolved into two components, one perpendicular, and the other parallel to the string. The first maintains an oscillation having the same frequency as that of the fork, and the secomd maintains an oscillation having half that frequency. The two oscillations occur, or can be made to occur in perpendicular planes.

[2] A parabolic arc is one of the Lissajons figures for the 1:2 composition ratio.

published (Fig. 2) gives only a very feeble idea of the effect. It can be seen from the shading that the paths of points on the string are curved arcs, and that the curvatures are in opposite directions in the two halves of the loop (which appear unequal in the photograph as the string pointed towards the lens of the camera).

7. For the study of the small motion at the nodes of the oscillation excited by transverse obligatory motion, it is necessary that the tension of the string be adjusted so that maintenance is as vigorous as possible. When this is done, it is noticed that points on the string (except near the nodes) describe 8 curves. The curve at the node, *i. e.*, the path compounded of the small motion at the node, and the large motion of half the frequency perpendicular to it, in neither an 8 curve not a straight line, but is a flat parabolic arc. From this, the phase-difference is again seen to be $\pi/2$. The direction of the curvature of the arc, in other words, the sign of the phase-difference, was found to agree with theory.

8. The above relates to the small motion at *any* node. One particular case is of importance, as it admits of independent experimental verification. For a string to be maintained in vigorous vibration by the imposition of an obligatory motion at one point, this point should itself lie at or near a node of the oscillation. It follows that under such circumstances, the imposed obligatory motion and the general oscillation of the string should differ in phase, the difference being equal to $\pi/2$ when the obligatory motion is imposed exactly at a node. This difference of phase between the motion of the prong and the general oscillation of the string, which may have been anticipated from the general principles of resonance, may be verified experimentally in two ways: (1) By stroboscopic observation and (2) by a tilting-mirror and Lissajous's figure arrangement. Before entering into experimental details regarding these, we may first discuss the mathematics of the questions dealt with above.

Mathematical Note.

9. The result of the investigation by Donkin of the problem of forced oscillations of stretched strings (Acoustics, 2d ed., pp. 121–124) is erroneous. Starting with the assumption that the obligatory

motion at the point $x = b$ is $p \sin nt + q \cos nt$ and taking dissipation of energy into account, the final approximate result obtained by him for the motion of points not near a node is

$$\sin \theta(p \sin nt + q \cos nt)/(\sin^2 \varphi + \delta_0^2 \cos^2 \varphi)^{\frac{1}{2}}.$$

From the original, it will be seen that $\sin \theta$ and $(\sin^2 \varphi + \delta_0^2 \cos^2 \varphi)$ do not involve the time t; the phase of the general motion of the string should therefore be identical with that of the obligatory motion at the point $x = b$. This result does not agree with that given in paragraph 8 above. The exact step in the mathematical work which introduces the error is putting $\tan \Phi = q/\rho$, where

$$\tan \Phi = (q\sigma_0 \sin \varphi + \rho\delta_0 \cos \varphi)/(\rho\sigma_0 \sin \varphi - q\delta_0 \cos \varphi).$$

To show that this step is erroneous, we may, without loss of generality, put $q = 0$. Then

$$\tan \Phi = \frac{\delta_0}{\sigma_0} \cot \varphi = \beta\varphi \cot \varphi = \frac{c\varphi}{2n} \cot \varphi.$$

Donkin's approximation is therefore equivalent to putting the damping factor $c = 0$. This is inadmissible, for the coefficient of the term, *i. e.*, $\cot \varphi$, is very large, and when $\Phi = i\pi$ at the exact stage of resonance, becomes infinite. At this stage $\tan \Phi = \infty$ and $\Phi = \pi/2$, whereas Donkin would have $\tan \Phi = 0$ and therefore $\Phi = 0$.

10. To compare the facts stated in paragraphs 1 to 8 above with the results of theory, we may make use of the notation and results given on pages 197–199 of Lord Rayleigh's Theory of Sound, Vol. I., second edition. The expression for the displacement at every point of the string maintained in vibration there given is

$$\gamma \frac{R_x}{R_b} \cos (pt + \Sigma_x - \Sigma_b),$$

where

$$R_x^2 = \sin^2 \alpha x + \frac{k^2 x^2}{4a^2} \cos^2 \alpha x$$

and

$$\tan \Sigma_x = \frac{e^{\beta x} - e^{-\beta x}}{e^{\beta x} + e^{-\beta x}} \cot \alpha x.$$

corresponding to an obligatory motion $\gamma \cos pt$ at the point $x = b$. In this expression β is a small quantity, and therefore

$$\tan \Sigma_x = \beta x \cot \alpha x,$$

and

$$(\Sigma_x - \Sigma_b) = \tan^{-1}(\beta x \cot \alpha x) - \tan^{-1}(\beta b \cot \alpha b).$$

This value of $(\Sigma_x - \Sigma_b)$ is very small and may be put equal to zero except in the two cases, where cot αx or cot αb is very large, *i. e.*, αx or αb is nearly equal to any integral multiple of π. In other words, the motion at any point is in the same phase as the obligatory motion unless it happens that (1) the point of observation, or (2) the point at which the obligatory motion is imposed, or (3) both points either coincide with, or are situated near nodes of the forced oscillation. In short, it may be said that there is a localized change of phase at the nodes, the difference between the phase at the node and at some point a considerable distance away from it being $\pi/2$.

11. The expression for the displacement can be written in the form

$$\frac{\gamma}{R_b}\left(\sin \alpha x \cos pT + \frac{kx}{2a}\cos \alpha x \sin pT\right),$$

which is seen to be of the type given in paragraph 4 above.

12. We now return to the experimental details referred to at the conclusion of paragraph 8 above.

Method (1).—A short length of the vibrating string is brightly illuminated, and an image of it is focused in the field of view of a stroboscopic disc. A slit held parallel to the string at some distance from it is illuminated, and the light issuing thereform suffers reflection at a small mirror attached to the prong of the electrically-maintained tuning-fork which keeps the string in vibration, and is then focused by a second lens into a linear image. The two images are adjusted so as to be in juxtaposition. On starting the tuning-fork and the stroboscopic disc, it can be seen that the two linear images are in different phases of motion, and the gradual change of the difference with the alteration of the tension of the string can be studied.

Method (*2*).—The motion of the prong and that of any point on the string are parallel to each other. To apply the method of Lissajous's figures for the observation of the phase-difference, a tilting-mirror arrangement was adopted. The tuning-fork actuates a light pivoted mirror by means of a thread which is kept taut by a spring pressing against the mirror. The plane of oscillation of the tilting-mirror is perpendicular to that of the vibration of the string, and a point on the latter brightly illuminated throughout its path with the aid of a cylindrical lens, and viewed by reflection, first at a fixed mirror and then at the tilting-mirror, is seen to describe a Lissajous's figure (circle, ellipse, or straight-line). From this figure the phase-difference can be inferred at once. It was found that the phase-difference was not quite independent of the amplitude of oscillation of the prongs: the explanation of this effect probably being that a large amplitude of oscillation involves a departure from constancy of the tension of the string, the average tension being greater than the normal value.

NOTE ON CROVA'S METHOD OF HETEROCHROMATIC PHOTOMETRY.

BY HERBERT E. IVES.

CROVA[1] suggested as a method of comparing the intensities of different colored lights, a comparison of their intensities at a single wave-length. By this means the actual photometry is reduced to the photometry of lights of the same color. For its utility and accuracy this method depends on the validity of a certain assumption, namely that there can be found a wave-length at which the relative energy emission in different illuminants is proportional to their total light. Such a wave-length may be looked for near the brightest part of the spectrum, since, owing to the very pronounced maximum of visual sensibility near .545μ and its rapid drop in value on either side, this point of maximum brightness is not greatly different for all the common incandescent illuminants. Obviously the method can only be applicable to light sources possessing continuous spectra, such as the black body, and sources whose energy distribution is not greatly different from the black body.

Crova found experimentally that he could measure the relative illumination from sunlight and from a Carcel lamp by a determination of the relative intensity at wave-length .582μ. It has been commonly assumed that this wave-length would also serve for the comparison of any illuminants lying between these two in color, either with one of these or with each other.

Lord Rayleigh[2] made a somewhat similar suggestion in a proposal to measure incandescent lamps with a "monochromatic telescope" using spectral yellow light. Rasch[3] and Lucas[4] have discussed the close relationhsip between the total light and the radiation

[1] Comptes Rendus, 93, p. 512.
[2] Phil. Mag., June, 1885.
[3] Ann. d. Physik, 14, pp. 193–203, 1904.
[4] Phys. Zeit., 6, 19–20, Jan. 1, 1906.

at a single wave-length, and the latter has deduced from Lummer and Kurlbaum's data on the luminous intensity of a black body at various temperatures the wave-length .542μ as the wave-length equivalent to total light in its behavior. Nernst[1] has used the Wien equation for the energy at one wave-length to represent total light in extrapolating from photometric observations on a black body. He obtained the values of his constants from observations between 1500° and 2300° and extrapolated to 2700°. Fery and Cheneveau[2] have derived equations connection the power expended and the candlepower of incandescent lamps, by assuming the candlepower to be proportional to monochromatic radiation following the Wien equation

$$I = Ae^{-\frac{B}{T}}.$$

All of these are applications of the original idea of Crova.

While this method of heterochromatic photometry has not been very widely used, partly because of its limitation to continuous spectra and partly because of practical difficulties, it is nevertheless of considerable experimental and theoretic interest. Experimentally it offers a means of making photometric readings where the color of the measured illuminant is changed through a wide range under conditions which do not permit of many readings or the use of calibrated compensating colored glasses. Such for instance would be measurements on the relation between power consumption and candlepower in incandescent lamps as the measurements are carried near the burning out temperature. Obviously, also, color-blind observers can make heterochromatic comparisons with the same accuracy as can observers of normal vision, provided their observations are made at the proper wave-length as determined from the average of normal individuals. On the theoretical side, use of Crova's method makes possible the calculation of light-power relations which would be prohibitively complicated were it necessary to include the complete visibility function connecting light with radiation.

In connection with a problem on the efficiency of incandescent

[1] Phys. Zeit., 7, pp. 380–383, June 1, 1906.

[2] Bull. de la Societe Internationale des Electriciens, 1909, p. 655.

solid radiation, in which incandescent lamps are to be employed, the writer has found it convenient to make use of Crova's method for practically the reasons outlined above. It seemed advisable before so doing to investigate the range within which the method would be valid; and from this investigation it appears that certain of the assumptions commonly made in its use are not entirely correct.

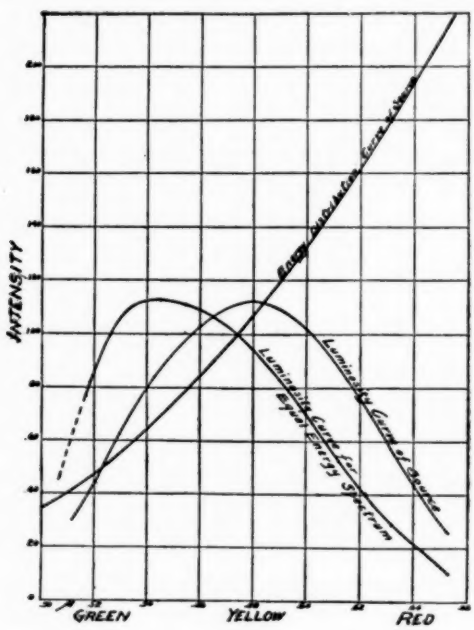


Fig. 1.

One assumption either stated or tacitly made is that the equivalent wave-length is that corresponding to the maximum of visual sensibility, *i. e.*, the wave-length of maximum luminosity for an equal energy spectrum. Another assumption—or rather, another practice—is to use the equivalent wave-length idea over a large range of illuminant colors (such as black body temperatures) without raising any question as to the limits within which the method holds, or making any determination of the order of magnitude of the errors within that range. This latter is involved in the belief that the wave-length which Crova derived from comparison of sunlight and the Carcel would hold for all intermediate illuminants.

The error in the first assumption is most clearly indicated diagrammatically, as in Fig. 1. Here is given the actual luminosity curve of a source (tungsten lamp through prism, etc.) and on the same drawing the luminosity curve for a spectrum of equal energy at all wave-lengths.[1] The latter is the product of the luminosity values for the source by the reciprocals of the energy values. The

[1] Data taken from work shortly to be published.

curves are drawn to be approximately of the same area, and, on the assumption (to be discussed below) that these areas are proportional to total light, it is at once evident that the wave-length of maximum sensibility, .545μ, does not give the relative light values, the error being about twenty per cent. The relative areas of the curves are actually given by the energy values at about wave-length .565μ. In other words the equivalent wave-length lies somewhere between the maxima of luminosity of the sources under comparison, and the wave-length of maximum sensibility does not enter in.

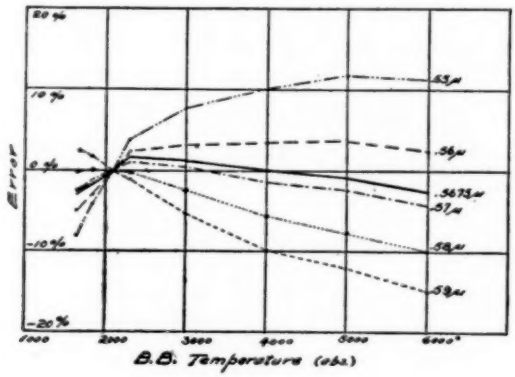


Fig. 2.

With regard to the second assumption, Fig. 2 gives in graphical orm some calculations on its validity in the case of the black body. The method of obtaining these data is as follows: Black body energy emission curves were calculated from the Wien equation; these were then each multiplied by the luminosity values corresponding to the spectral wave-lengths. These values were taken from a recently determined normal luminosity curve, from observations by five observers at a moderately high illumination. The areas of the resultant curves (the type of which is exhibited in Fig. 1) are taken as proportional to the total light, as measured on a photometer. A word as to the assumption involved here. Let the energy per unit of spectrum length at λ be W_λ, then the total energy in the spectrum is

$$W = \int_0^\infty W_\lambda d\lambda.$$

Now the light values, or relative luminosities of the different wave-lengths, are different, as shown by the luminosity curve. To each wave-length we may ascribe a factor K_λ, its "light va'ue," which is zero outside the visible spectrum; then the assumption involved in the above process is, concisely stated, that the total light L is equal to

$$\int_0^\infty K_\lambda W_\lambda d\lambda.$$

The validity of this assumption has never been seriously questioned. Experimentally it has received verification by Abney and by Tufts, and has been used by others without introducing any observed errors. While recent work by the writer would indicate the necessity of a more searching inquiry as to its validity under all conditions there can be practically no doubt that this process of integration represents the truth to a fair degree of approximation.

Having in this manner determined the relative light values, the relative values for the energy at various wave-lengths were read off the energy distribution curves. A table was then made out, taking the light and energy values at 2100° abs. arbitrarily as the unit, and the changes in total light with temperature were compared with the changes in the emission at various wave-lengths in the spectrum. Fig. 2 gives the errors introduced by limiting the photometry to light of one wave-length, where the standard of intensity is the black body at 2100°. When the measured light is the same in color as the standard, any wave-length may be employed, hence the error at this point is zero.

Several facts are here brought out. First, no one wave-length can be used as an equivalent wave-length for more than a certain definite range of temperature, for any given allowable error. Wave-length .58μ for instance would be selected to compare our standard with a black body at any temperature between 1600° and 2300° with an error of not more than .1 per cent., but if a black body at a higher temperature is measured an increasing error is introduced, until at 6000° it amounts to fifteen per cent. The horizontal portion of each curve shows the temperature range through which the corresponding wave-length may be used as an equivalent to total light. Between 1600° and 2300°, .58μ is indicated; between

3000° and 4000°, .56μ; between 4500° and 5500°, .55μ. A wave-length which would serve for comparing black bodies at 2000° and 5000° would introduce errors of several per cent. if intermediate points were compared with either extreme.

In the second place it appears that the wave-length of maximum visual sensibility—in this case .545μ—is not the equivalent one for any black body comparisons except between 5000° and 6000°; that is, far above the temperatures represented by any incandescent lamp, or other artificial illuminant of the incandescent solid type.

Wave-length .5675μ has been picked out by trial as the one which could be employed for an equivalent wave-length over the longest range with the least error. The error with this wave-length is less than three per cent. in either direction. If, however, the method is to be used only within the limits of temperature represented by the ordinary incandescent lamps—namely 1600° to 2400° abs.—wave-length .58μ should be chosen.

In order to obtain a check on these calculations measurements were made on a tantalum lamp over a wide range of voltages, both of the total light and of the monochromatic light of various colors. For this purpose a simple Martens equality photometer was used, from which the eye-piece could be removed and a spectrometer substituted. Over the slit of the spectrometer was placed a lens to form an image of the photometer field in a convenient place in the optical system of the instrument. Observation was made through an eye-slit at the point where the spectrum is formed. Under these conditions the photometer field is seen in monochromatic light of the wave-length incident upon the eye. The tantalum lamp was operated at various voltages between 65 and 115, its normal voltage being 100. The highest voltage makes the color about that of a normally operated tungsten lamp while the lowest is about like the Hefner. A "4-watt" carbon lamp was used as comparison standard, the tantalum lamp matching it in color at 78 volts. Photometric observations for total light were made by two observers who possess very similar color vision, both of their maxima of sensibility for high illumination falling at .54μ. Their agreement in reading was excellent, the average difference being

0.2 per cent. and showing no systematic drift with increased color difference. The error of measurement with monochromatic light was larger than with the total because of the smaller amount of light available, the error amounting probably to as much as one

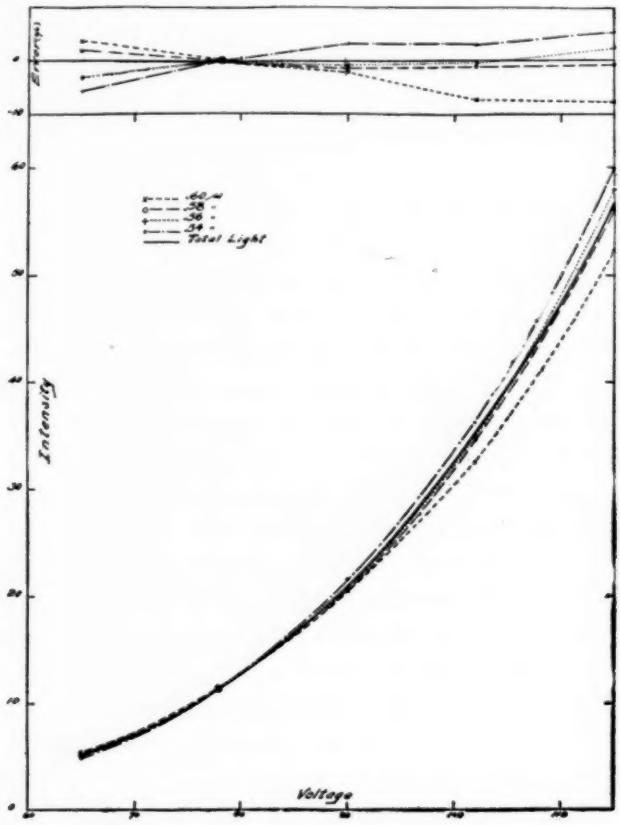


Fig. 3.

or two per cent. A larger number of measurements would have reduced the uncertainty, but those obtained (average of five settings) were so decisive as to make further experiment superfluous.

In Fig. 3 are plotted the experimental results. The full curve is the total light curve. The various dotted and dashed lines exhibit the monochromatic light curves, as designated in the key. At the top are plotted the percentage errors. At the voltage of

color match any wave-length may be chosen, but it is at once evident that as the color of the two lamps (standard and test lamp) become different, the relative candle powers are not followed equally well by all monochromatic radiations. The equivalent wave-length lies between .56μ and .58μ. The data of Fig. 2 indicate .58μ for this range of colors, but the two observers in this experiment have their maxima of sensibility further toward the blue than the mean of the five taken as normal in Fig. 2. It is seen that wave-length .54μ, the maximum of visual sensibility of the present observers, does not represent the total light change. The total error from end to end amounts in fact with this wave-length to about twelve per cent. In short, the results derived from calculation are very satisfactorily confirmed.

The result of the investigation is therefore to show that the oft-repeated statement that the equivalent wave-length is the wave-length of maximum sensibility is in error: and that any equivalent wave-length is such to a given degree of accuracy for but a limited range of black body temperatures. The equivalent wave-lengths for various black-body temperatures, and the one holding over the longest range with the least error for normal eyes, have been indicated.

The writer takes pleasure in acknowledging his indebtedness to Mr. M. Luckiesh for assistance in making the observations here given and for the preparation of the drawings.

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CLEVELAND, OHIO.

A NEW METHOD OF PRODUCING RIPPLES. OPTICAL ANALOGIES.

BY A. H. PFUND.

THE use of ripples in illustrating optical phenomena is too well established to require justification. Since the publication of Vincent's[1] work on mercury ripples, little has been or can be added to the discussion of the application of these phenomena to experiments in optics. In view of their recognized pedagogical value, it seems strange that but little use is made of ripple experiments in lecture-room demonstration. The reason for this becomes apparent, however, when the complexity of the apparatus, necessary for producing ripples and making them visible, is taken into con-

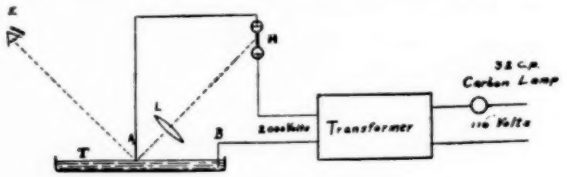

Fig. 1.

sideration. Recently I observed a phenomenon which was used as the essential feature in the construction of a ripple apparatus of great simplicity. The following diagram (Fig. 1) shows the complete apparatus.

The current from the secondary of a small transformer (2,000 volts, 60 cycles per second) passes through a helium vacuum tube H, then through the wire A (0.5 mm. diameter) into distilled water contained in the hard rubber tray T and finally back through the wire B to the transformer. While the wire B dips deeply into the water, the wire A just touches the surface—and it is at this point that the ripples are produced. The fact that the phenomenon is

[1] J. H. Vincent, Phil. Mag., 1897, 43, p. 417; 1898, 45, p. 191. H. M. Reese, Ap. Jnl., 1906, 24, p. 47.

Fig. 3.

Fig. 4.

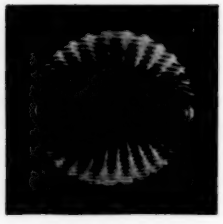

Fig. 5.

Fig. 6.

Fig. 7.

A. H. PFUND.

observed only when such poorly conducting liquids as distilled water and alcohol are used, indicates that the ripples are produced by the effect of the large potential drop at the point of contact. No mechanical vibration of the wire A is present.

Each pulsation of the current produces a ripple which moves out from its source with so great a velocity that, in continuous illumination, the surface of the water appears undisturbed. However, by making the illumination intermittent, *i. e.*, synchronous with the rate of production of ripples (which is accomplished by the use of the vacuum tube H), the ripples apparently stand still and may be observed at leisure. For observing or photographing ripple-phenomena, the lens L is placed as indicated in the diagram

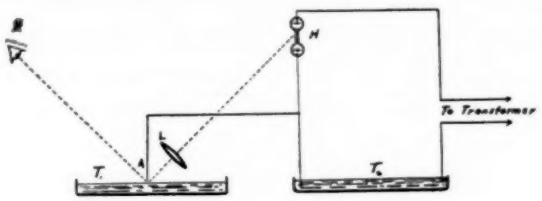

Fig. 2.

where the helium tube and the eye or camera lens are at conjugate foci. So as to obtain light from a "point" source, all light coming from the helium tube, with the exception of that coming from 1 mm. of the capillary, was screened off.

That the ripples are produced by electrostatic effects is further proved by the effects observed with the apparatus arranged according to diagram shown in Fig. 2. Here the current passes through the vacuum tube H and then through distilled water in tray T_2 (this water merely serves as a high resistance). The end of the wire A does not touch the surface of the water in tray T_1, but is brought as near it as possible without making contact. The eye placed at E sees beautiful ripples having A as their center.

The proper contact, necessary for giving the phenomenon at its best must be determined by trial. If the lower end of the wire A (Fig. 1) is too high above the general surface of the water, the ripples are too violent; if, on the other hand, the wire dips down into the water too deeply, the ripples are too feeble.

In order to avoid annoying reflections at the sides of the containing vessel, it is necessary to remove all traces of grease so that the water may be drawn up slightly where it meets the vessel. If, however, reflections are desired, a piece of glass or sheet-metal, covered with grease is placed into the water.

In explanation of the photographs it might be stated that, as a consequence of viewing the water-surface at an angle of about 45°, the reflected image of the wire *A* is always visible and the otherwise circular ripples appear elliptical. Whenever any object is placed in the water, the planarity of the surface is destroyed and, in consequence, a dark region of considerable extent is produced—making it difficult to determine the character of the object from the photograph. Whenever necessary, I have indicated with dotted lines the position of the object in question. The photographs here presented show only a few of the results which may be obtained with this particular arrangement of apparatus. Many modifications suggest themselves for showing more complicated phenomena, but, as these advantages are gained at the expense of simplicity, I have decided not to discuss them at present.

Fig. 3. This illustrates the character of ripples from a single point source.

Fig. 4. This illustrates Young's interference experiment. Two point sources, vibrating in phase, give rise to interference hands whose loci in space are confocal hyperbolæ of revolution (the sources lying at the foci). The existence of hyperbolæ is well shown in the photographs. To obtain these results, the current is led into the water by both wires.

Fig. 5. This shows the same phenomenon with wide separation of sources. The narrowness of the interference bands, as contrasted with the preceding case, is to be noted.

Fig. 6. The analogy of Lloyd's mirror is here shown. A greased piece of brass, at a distance of 1 cm. from the source, acts as a plane mirror—the incident and reflected waves interfering to show well-marked interference bands. As is well known, the waves leaving the reflector seem to come from a source situated as far behind the mirror as the real source is in front of it. As a result, interference bands of the same type as those shown in Figs. 4 and 5 are

produced. The reflector simply rests in the water and is not electrically connected.

Fig. 7. Here the ripples are produced by a point source in front of an obstacle and the phenomenon within the gemoetrical shadow is photographed. In the illustration it is first to be observed that the ripples rapidly enter the region of the geometrical shadow (diffraction). These diffracted waves, coming from the two edges of the obstacle, produce an interference pattern which could be duplicated by two real sources—one at each corner of the obstacle. This becomes evident from the hyperbolical form of the fringes. The straight central fringe (x) represents the analogy of Poisson's famous experiment.

Of the many other experiments which I have carried out with this apparatus, the one illustrating refraction is worthy of particular notice. This phenomenon was observed by floating a drop of machine oil on the surface of the water. The drop slowly expands into a circular disc on whose surface ripples are observed. Since the surface-tension of oil is much less than that of water, the velocity of the ripples is less on the oil surface, hence the waves are refracted. The phenomenon, though readily observable, was not photographed for the reason that during the necessary time of exposure (three minutes) the drop had expanded to such an extent that it covered the entire surface of the water.

In conclusion I wish to repeat that the only novelty claimed for these experiments lies in the simplicity of the method of producing ripples and making them visible. If this discussion will help in bringing about a more widespread use of ripples in the elucidation of optical principles, I shall consider my purpose in writing this note well accomplished.

JOHNS HOPKINS UNIVERSITY,
October, 1910.

EFFECT ON THE CATHODE FALL IN GASES PRODUCED BY THE EVOLUTION OF GAS FROM THE CATHODE.

BY L. A. JONES.

IT has been shown by Skinner[1] that a fresh metal electrode when used as cathode in a glow current in helium and argon gives off hydrogen at the rate required by Faraday's law for electrolytes. This effect was found to continue only for a few minutes after starting the current and then only when fresh metals were used. In an atmosphere of hydrogen no increase in pressure was observed, indicating either that no gas was given off by the cathode or that the anode absorbed gas at the same rate as the cathode evolved it. Experiments performed shortly afterward[2] with a nitrogen atmosphere gave similar results.

Holman first[3] and Chrisler,[4] more extensively, investigated the effect of absorbed hydrogen on the photo-electric activity of metals. Chrisler showed conclusively that absorbed hydrogen is an important factor in the photo-electric current. He found that use as cathode (which reduces the supply of hydrogen in the metal) always decreases the photo-electric effect; while use as anode in hydrogen or simply standing in hydrogen (which charges the metal with the gas) increases the photo-electric effect enormously.

Later Chrisler[5] proved by direct measurements the absorption of gas by the anode. He found that silver, mercury and the alkali metals when used as anode absorb hydrogen at the rate required by Faraday's law for electrolytes. These metals, with the exception of mercury, were also found to absorb nitrogen at the same rate. In some exceptional cases helium also was absorbed.

[1] C. A. Skinner, PHYS. REV., vol. 21, p. 1; Phil. Mag., 6, vol. 12, p. 481, 1906.
[2] C. A. Skinner, PHYS. REV., vol. 21, p. 169.
[3] W. F. Holman, PHYS. REV., vol. 25, p. 81.
[4] V. L. Chrisler, PHYS. REV., vol. 27, p. 267.
[5] V. L. Chrisler, PHYS. REV., vol. 24, p. 461.

Whiddington[1] investigated the effect on the emitted cathode rays arising from a treatment of the metal similar to that given by Chrisler in the photo-electric effect. He found that an electrode after having been used for some time as cathode emitted more homogeneous and slower moving cathode rays than were obtained when the electrode was fresh. On the other hand just after use as anode a flash of intense radiation was observed. Further, with a glow current through air using an aluminium cathode, he found the cathode fall increased rapidly to a maximum value in about three minutes after starting the current, then fell gradually to a final constant value which was lower than the initial.

The object of the present investigation was to make a more extended study of the effect which the evolution of gas by the cathode produces on the cathode fall.

The investigation was divided into two parts; the first dealing with the effect on the cathode fall under a constant current density; the second, with the effect on the "normal" cathode fall.

I. The Effect with Constant Current Density.

The plan followed was to use a discharge tube so constructed that after a cathode had been used for some definite time a fresh one of the same metal could be substituted for it immediately without changing any other conditions. This scheme eliminated any effect arising from a change in the gas, and gave, by comparing the final value of the old cathode with the initial value of the new one, a reliable indication of the effect of use on the metal alone.

Apparatus.

A vertical section of the discharge tube is shown in Fig. 1. The discharge chamber M (diam. 3 cm., length 20 cm.) terminates below in a large cylindrical one, the axis of which is parallel to the axis of the tube M. The lower chamber is about 12 cm. in diameter and 3 cm. deep. The entire discharge tube is of glass; all permanent joints, that could not be fused, being cemented with water glass and covered on the outside with a coating of de Khotinsky's laboratory cement. Inside the lower chamber is fitted a circular glass

[1] R. Whiddington, Proc. Camb. Phil. Soc., vol. 15, p. 183.

plate, slightly less in diameter than the chamber, capable of carrying ten cathodes, E, attached to the plate by means of hard rubber nuts, K. This circular plate can be rotated about its axis by means of a key introduced through the ground joint, J, thus allowing any cathode to be brought into position at the lower end of the discharge chamber. Tin-foil strips lead from the cathodes to small brass rivets, C, coming up through a hard rubber disk, D, fixed to the circular plate. These rivets make contact with a brass brush, B, connected to a brass collar which in turn is connected to a platinum wire, F, sealed through the wall of the tube. Thus all of the cathodes except the one directly beneath the discharge chamber are insulated from the circuit, but any one of them can be brought into position by simply turning the ground joint, J. A glass plate properly cut covered the tin-foil strips and fitted up to the rims of the cathodes. As a means of measuring the cathode fall a wire, W, was introduced through the ground joint, N. This probe is of fine aluminium wire sheathed with glass to within about 5 mm. of its inner end. It is bent into a form such that by simply turning the ground joint the distance between the probe and the cathode can be varied from three tenths to two centimeters. In order to change or repolish the cathodes it was necessary to take off the base plate, A, which allowed the circular plate, carrying the

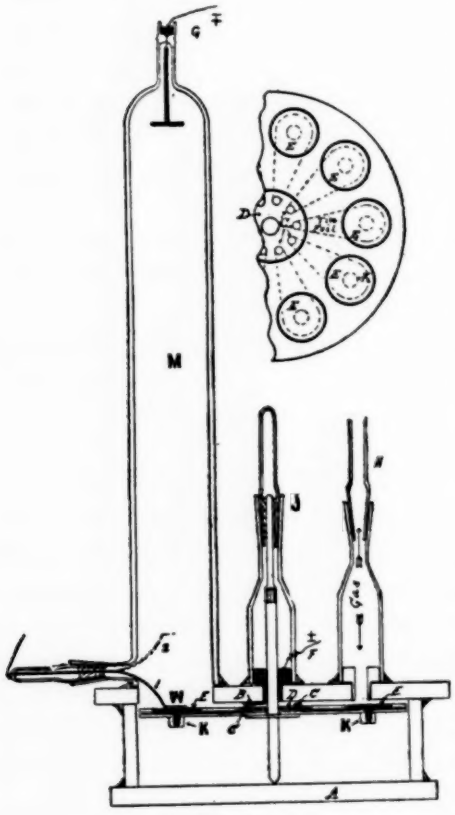

Fig. 1.

cathodes, to be removed. The base plate was therefore sealed on with de Khotinsky's laboratory cement which was applied to the outside only. An aluminium anode of very small area (to prevent appreciable absorption of gas) was introduced at *G*. Connection to the evacuating and drying system was made through the ground joint, *H*.

The cathode fall with various metals was measured in both hydrogen and helium. The hydrogen was obtained from aluminium in a potassium hydrate solution, dried carefully with P_2O_5, and then stored in a glass bulb connected to the evacuating system by a ground joint.

The helium used was obtained in sealed tubes from Messrs. Thomas Tyrer and Co., London, and transferred to a glass bulb that had been carefully dried and evacuated for several days. This bulb was also connected to the system by a ground joint.

All ground joints and stop-cocks included in the evacuating, drying and gas-supply system were lubricated with a mixture free from volatile hydrocarbons.

Gas-pressure measurements were made by means of a McLeod gauge which magnified the pressure one hundred times. Evacuation was produced by a Barr and Stround mercury pump.

The cathode fall was measured by a Kelvin quadrant-electrometer, a definite potential being placed between the quadrants, and the cathode fall between the needle and one pair of quadrants. The instrument in this form was calibrated by comparison with a Weston voltmeter.

The electric current was furnished by a battery of six hundred small storage cells, was measured by a Weston milliammeter, and regulated by means of a resistance of cadmium iodide in amyl alcohol. The electrometer was protected by a similar resistance tube introduced in the line leading to the needle.

The cathodes used were 1.5 cm. in diameter and about 1 mm. thick. They were polished to a mirror surface with infusorial earth mixed with tallow, and finally with clean new felt and new cotton cloth.

Experimental Results.

Platinum, zinc, aluminium and magnesium cathodes were tested in both gases.

The method followed was to evacuate the system (approximate volume 580 c.c.) and then after flushing with the gas to be used, to admit sufficient gas to bring the pressure to the desired value. The working limits of gas pressure were determined by making preliminary tests. The pressures used were such that the negative glow entirely covered the cathode; the Crooke's dark space varying in thickness from 1 mm. to 5 mm.

Gas having been admitted to the desired pressure the current was started, quickly adjusted to a definite value (one milliampere in all cases) and readings of gas-pressure and cathode-fall taken at regular time intervals (every 1.5 minutes in some cases, every two minutes in others).

The probe was placed in the negative glow so that a movement of 2 mm. or more, depending on the gas pressure, toward or away from the cathode caused no change in the electrometer reading. This adjustment showed that the fall of potential observed was the desired "cathode fall."

After taking readings for some time (10 to 40 minutes) a new cathode of the same metal was brought into position and its fall immediately measured. As already stated this mode of procedure was followed to eliminate variations arising from a change in the pressure or composition of the gas. Any variation in the cathode fall arising from a change in the cathode itself, was thus indicated by a difference in the electrometer reading the moment a fresh cathode was brought in.

The following tables give the results of tests with the four different metals (aluminium, zinc, platinum and magnesium) in both hydrogen and helium.

Table I. gives in detail the results obtained from an aluminium cathode in helium, observations of both gas pressure and cathode fall having been made at intervals of 1.5 minutes. It shows a gradual increase in gas pressure (2.74 mm. to 3.11 mm.) accompanied by a decrease in cathode fall (199.5 to 191 volts). When the fresh cathode was introduced however its fall was found only one volt less than the old (191 to 190 volts), the gas pressure having risen while measuring the fall of the new cathode from 3.11 mm. to 3.14 mm. This difference of one volt is too great to be attributed to

TABLE I.

Aluminium in Helium. Current Density: .56 (ma./cm.).

Duration of Current (min.)	Gas Pressure (mm.).	Cathode Fall (Volts).	Duration of Current (min.)	Gas Pressure (mm.).	Cathode Fall (Volts).
0	2.74	199.5	13.5	3.00	193.0
1.5	2.79	197.0	15.0	3.02	192.5
3.0	2.84	196.5	16.5	3.04	192.0
4.5	2.87	195.5	18.0	3.05	192.0
6.0	2.90	195.0	17.5	3.06	191.5
7.5	2.93	194.5	21.0	3.08	191.5
9.0	2.94	194.5	22.5	3.10	191.5
10.5	2.96	193.0	24.0	3.11	191.0
12.0	2.98	193.0			
				Fresh Cathode.	
				3.14	190.0

TABLE II.

Cathode Fall of Depleted and of Fresh Metals in Helium. Current Density: .56 ma./cm².

Cathode Metal.	Sample Tested for Duration.			Fresh Sample in same Gas.	
	Duration of Curr. (min.).	Gas Press. (mm.).	Cathode Fall (Volts).	Gas Press.	Cathode Fall.
Aluminium.	0	.89	255	—	—
	20	1.16	238	1.19	235
	0	2.74	196	—	—
	9	2.99	194	3.03	193
	0	2.04	214	—	—
	39	2.58	198	2.60	199
Magnesium.	0	.83	294	—	—
	20	1.11	270	1.13	273
	0	2.20	214	—	—
	16.5	2.63	207	2.69	204
Zinc.	0	1.94	291	—	—
	24	2.31	286	2.34	285
	0	1 02	377	—	—
	25	1.33	355	1.33	356
	0	2.83	262	—	—
	33	3.28	255	3.31	253
Platinum.	0	.82	408	—	—
	20	.89	403	.89	402
	0	1.85	214	—	—
	12	1.87	214	1.88	214

the change in gas pressure between the final observation on the old and that on the new cathode, but it is less than the difference in fall obtained under identical conditions with different fresh cathodes. On the other hand the difference between the initial and final observations with the first cathode (8.5 volts) is large in comparison with the change produced by the introduction of the fresh cathode, hence it is to be concluded that the observed variation in cathode fall with use was caused not by any change in the cathode itself, but rather by the change in condition of the gas content of the discharge tube.

In Table II. are given condensed results of similar tests with other metals in which the readings with the first cathode are followed by the observation taken immediately after the introduction of the second cathode of the same metal. In each case after the

TABLE III.

Cathode Fall of Depleted and of Fresh Metals in Hydrogen. Current density: .56 ma./cm².

Cathode Metal.	Sample Tested for Duration.			Fresh Sample in Same Gas.	
	Duration of Curr. (min.).	Gas Press. (mm.).	Cathode Fall (Volts).	Gas Press.	Cathode Fall.
Aluminium.	0	.82	408	—	—
	20	.89	403	.89	402
	0	1.91	241	—	—
	15	2.10	236	2.11	236
	0	.96	305	—	—
	20	1.28	285	1.28	288
Magnesium.	0	1.76	244	—	—
	15	2.06	234	2.11	231
	0	2.11	231	—	—
	12	2.35	227	2.38	228
Zinc.	0	.90	437	—	—
	22	1.29	406	1.32	404
	0	1.40	388	—	—
	15	1.40	386	1.40	385
Platinum.	0	.96	540	—	—
	20	.94	545	.94	550
	0	1.68	310	—	—
	21	1.83	314	1.85	314

fresh cathode was introduced the fall was measured as quickly as possible, the time required for the deflection of the electrometer to become steady being about one minute.

From the table the maximum change in cathode fall, occurring when the fresh cathode was introduced, was 3 volts, the minimum 0, and the mean 1.6 volts. This difference between the old and the fresh metal was in most cases accompanied by a slight increase in gas pressure, but could not be ascribed to that since the change in fall was sometimes an increase, sometimes a decrease. Table III. gives similar results obtained with an atmosphere of hydrogen.

While therefore the results reveal a considerable decrease in cathode fall with use (the only exception occurring with platinum in helium) accompanied in general by an increase of gas pressure, yet they appear to prove conclusively that with a constant current density there is only a relatively small change in the cathode fall in either hydrogen or helium arising from a change taking place in the metal. The change in fall taking place when the fresh cathode was introduced, sometimes positive and sometimes negative, was always much smaller than the difference between the initial and final values obtained with the original cathode. The fact that in general a relatively large decrease in cathode fall occurred when there was also a relatively large increase in gas pressure suggests that the change might very reasonably be ascribed to the increase in pressure simply, though no tests were made to verify this assumption.

II. The Effect with Normal Current Density.

The foregoing tests with a constant current density (greater than the "normal") having shown no change in the cathode fall caused by the condition of the cathode itself, it remained to investigate the effect with the "normal" current density.

The plan adopted was to use a long cylindrical cathode and a gas pressure such that the negative glow did not entirely cover it, thus making sure that the normal current density was obtained.

The apparatus in this case was not so constructed as to permit the introduction of a fresh cathode without changing the gas.

Apparatus.

The apparatus with the exception of the discharge tube was the same as that used in the previous work. A diagram of this is given in Fig. 2. It was about 3 cm. in diameter and 20 cm. long. A

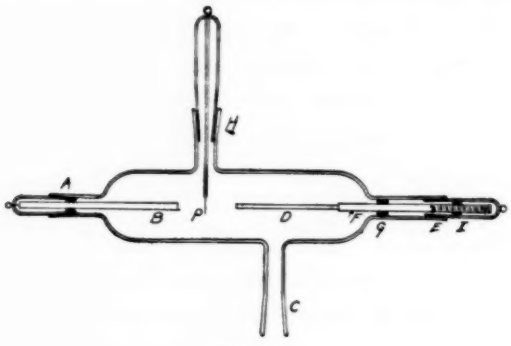

Fig. 2.

square brass rod *F*, threaded at one end, passing through the brass guide collar *G*, and a threaded nut *I*, carried the cathode *D*. By means of this arrangement the distance between the fixed probe, *P*, and the cathode could be readily varied. The probe was made of fine aluminium wire sheathed with glass to within 5 mm. of the end, and introduced through the ground joint, *H*. The anode, in this case, of rather large area, was introduced through the ground joint, *A*. Both anode and cathode were thus coincident with the axis of the tube, the probe crossing it at right angles. The ground joint, *C*, served to connect the tube with the rest of the system.

The cathodes used were of wire about 8 cm. long and of different diameters (varying from .5 mm. to 2 mm.). They were cleaned and polished in the same manner as before.

Experimental Results.

When the current was started the negative glow appeared at the end of the cathode, resembling in shape a small test-tube slipped over the end of the wire. The length of the cathode covered depended upon the kind of metal used, the kind of gas in the tube and the gas pressure. Conditions were always chosen so that the glow did not entirely cover the cathode, thus insuring normal current density.

In measuring the cathode fall, care was taken to see that the probe was in the negative glow, and so placed that the exposed portion was entirely in the region of zero potential gradient.

Observations on the fall were made at regular intervals (2 minutes). The pressure was observed at the beginning and end of the run, and at irregular intervals between.

The following tables give results of the tests, the first (Table IV.) in detail, the others (Tables V. and VI.) only the initial and final readings.

The same kinds of metal and gas were used as in the first part of the investigation. The magnitude of the current was one milliampere throughout.

TABLE IV.

Aluminium in Helium. Normal Current Density.

Duration of Current (min.)	Gas Pressure (mm.).	Cathode Fall (Volts).	Duration of Current (min.)	Gas Pressure (mm.).	Cathode Fall (Volts).
0	1.62	—	16	1.65	170
2	—	157	18	—	171
4	—	160	20	—	172
6	1.64	161	22	—	173
8	—	166	24	1.64	173
10	—	167	26	—	174
12	—	168	28	—	174
14	—	169	30	1.64	175

TABLE V.

Effect of Duration of Current on the Normal Cathode Fall in Helium.

Cathode Metal.	Duration of Current (min.).	Gas Pressure (mm.).	Cathode Fall (Volts).
Magnesium.	0	1.90	173
	34	1.93	192
Zinc.	0	1.69	170
	34	1.73	185
Platinum.	0	1.90	245
	36	1.90	249

The observations given in Table IV. (aluminium in helium) show that *the normal cathode fall increased* to a marked extent (157 volts to

175 volts) with duration of current while the gas pressure remained almost constant (1.62 to 1.64 mm.). The change here was therefore just the reverse of that noted in the first part.

Similar results were obtained in all other tests (Tables V. and VI.). The effect of duration of current was the same therefore, notwithstanding the fact that in two out of three tests made with helium the gas pressure increased slightly with duration of current, while with hydrogen (with but one exception) the pressure decreased.

TABLE VI.

Effect of Duration of Current on the Normal Cathode Fall in Hydrogen.

Cathode Metal.	Duration of Current (min.).	Gas Pressure (mm.).	Cathode Fall (Volts).
Aluminium.	0	.85	190
	31	1.16	217
	0	3.12	199
	30	3.01	211
Magnesium.	0	3.23	205
	26	3.03	217
Zinc.	0	3.24	220
	32	3.14	234
	0	1.97	227
	32	1.83	250
Platinum.	0	3.48	315
	20	3.16	331
	0	3.23	310
	30	3.23	363

In order to make certain that the observed change in fall did not arise from the slight change in gas pressure which took place, observations were made on one cathode (with very short duration of current in each case) with pressures varying through a wide range. From the results of these tests (given in Tables VII. and VIII.) it may be seen that while the normal fall, with but one exception, decreased slightly with a decreasing gas pressure a relatively large change in pressure caused only a small lowering of the fall as compared with that produced by duration of current.

The results given in Tables VII. and VIII. are interesting aside

TABLE VII.

Effect on Normal Cathode Fall in Helium Caused by Changing the Gas Pressure.

Cathode Metal.	With Same Gas at Decreasing Pressures.		With Fresh Gas.	
	Gas Pressure (mm.).	Cathode Fall (Volts).	Gas Pressure (mm.).	Cathode Fall (Volts).
Aluminium.	4.00	175		
	1.85	170		
	.87	168		
	.56	165	3.97	176
Magnesium.	4.33	191		
	2.97	190		
	1.96	188		
	1.04	186	4.32	196

TABLE VIII.

Effect on Normal Cathode Fall in Hydrogen Caused by Changing the Gas Pressure.

Cathode Metal.	With Same Gas at Decreasing Pressures.		With Fresh Gas.	
	Gas Pressure (mm.).	Cathode Fall (Volts).	Gas Pressure (mm.).	Cathode Fall (Volts).
Aluminium.	3.72	213		
	2.03	211		
	1.13	210	3.90	213
Zinc.	6.80	256		
	3.65	253		
	1.45	250	6.35	256
Platinum.	6.55	363		
	3.85	360		
	2.35	354	6.46	364

from the fact that they prove that the observed change in the normal cathode fall with duration of current is not caused by the change in gas pressure. For, they appear to contradict the generally accepted conclusion that the normal cathode fall is independent of the gas pressure.[1] The variation here evidently cannot arise from a change in the cathode with use, for with fresh gas at the original pressure the fall returned to its original value. However

[1] Warburg, Wied. Ann., 31, p. 545, 1887; 40, p. 1, 1890.

it may, as shown in the first part of this paper, arise from a change in the condition of the gas, since the sequence of observations were made by reducing the gas pressure and not by introducing fresh gas.

Summing up the results: (*a*) It has been found that use of a metal as cathode with a constant current density greater than the "normal" *does not change the metal* in a way such as to appreciably change its cathode fall, but it does in general *lower* the cathode fall by producing some *change in the gas*. (*b*) On the other hand, use of a metal as cathode with "normal" current density *increases* its cathode fall, which in light of (*a*) appears reasonably explicable only by assuming that in the first case the cathode fall and current density were large enough to keep the surface of the metal clean by "disintegration," but not in the second.

The writer wishes here to express his thanks to Dr. C. A. Skinner, of the University of Nebraska, for help and encouragement given so freely during the pursuit of this investigation.

BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA, LINCOLN,
June, 1910.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

TERMINAL VELOCITY OF FALL OF SMALL SPHERES IN AIR AT REDUCED PRESSURES.[1]

BY L. W. MCKEEHAN.

IN previous work by John Zeleny and the author[2] it was found that Stokes' law for the terminal velocity of fall of a small sphere in a viscous fluid is upheld in air at atmospheric pressure, for spheres differing in size and composition.

E. A. Cunningham has given a theory[3] in which the molecular structure of the gaseous medium is considered, and finds that the velocity calculated by Stokes' law should be multiplied by a factor,

$$\left[1 + 1.63\,\frac{l}{a}\left(\frac{1}{2-f}\right)\right],$$

where l is the mean free path of the gas molecule, a is the radius of the sphere, and f is the fraction of the total number of molecular impacts on the surface of the sphere, which are regularly reflected. The second term in this expression becomes important only for low pressures, or for very minute spheres at atmospheric pressure. An examination of the derivation of this factor leads to the conclusion that it should be written

$$\left[1 + .940\,\frac{l}{a}\left(\frac{4}{5-f}\right)\right].$$

Neither of these factors agrees numerically with experiments at pressures down to a fraction of a millimeter of mercury, for spheres of wax ranging

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 27–30, 1910.

[2] PHYS. REV., Vol. XXX., p. 535, May, 1910.

[3] Proc. Roy. Soc., Ser. A, Vol. 83, p. 357, 1910.

from $a = .003$ cm., to $a = .0002$ cm. An empirical factor,

$$\left[1 + k\frac{l}{a}\right],$$

is found, however, to represent with accuracy the results obtained.

UNIVERSITY OF MINNESOTA.

ON THE RESOLUTION OF THE SPECTRAL LINES OF MERCURY.[1]

BY J. C. MCLENNAN AND E. N. MACALLUM.

WITH this communication a series of slides were exhibited which illustrated the resolving power of a high grade echelon spectroscope recently made by The Adam Hilger Co. for the Physical Laboratory at Toronto. With this instrument the green line of mercury 5461 ÅU was shown to consist of a central doublet accompanied by three satellites of greater and by three of smaller wave-length, and the blue line 4359 ÅU to consist of a central strong line accompanied by three satellites of greater wave-length and by two of shorter wave-length. These results are in good agreement with the components of the same lines recently obtained by Gale and Lemon and others with a seven-inch Michelson plane grating.

Slides were also shown which illustrated the magnetic resolution of the satellite of shortest wave-length of the line 5461 ÅU. Under magnetic fields of 2,000 Gauss this line was resolved into a quartet, the inner doublet of which corresponded to vibrations parallel to the magnetic field, and the outer to vibrations perpendicular to the magnetic field. Measurements made on the displacements of the lines constituting the quartet showed that the magnetic separation of the two inner doublets corresponded approximately to three fourths and that of the outer components to three halves of a normal triplet.

In taking the photographs of the magnetic resolution of this satellite it was necessary to use a source of light of strong intensity, and even with such a source to give long exposures. The arrangement adopted is shown in Fig. 1.

A magnet was provided with pole pieces each of which was divided into two pieces that could be bolted together. The pole pieces were provided with a groove, and in this groove an ordinary Cooper Hewitt lamp was fastened as shown in the figure with its axes parallel to the direction of the field. With this tube light of strong intensity was provided and there was no danger of the tube being melted by the discharge for the effect of the field was to concentrate the discharge along the axis of the tube. With such an arrangement exposures of any duration could be used, and magnetic fields as high as 3,500 gauss could be obtained readily.

[1] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 28–31, 1910.

With this paper diagrams were also shown illustrating the dependence of the constitution of the mercury line 5461 ÅU upon the source of light.

With the Cooper Hewitt lamp no resolution of the main doublet could be observed but when a Hæreus quartz mercury lamp was used this line was clearly seen to be a doublet. A further peculiarity of the resolution of this line obtained with the quartz lamp was that after the lamp was in action a few minutes each of the lines of this main doublet and those corresponding to each of the satellites became somewhat diffuse and approximately of the same intensity and configuration. The general appearance of the resolved line with this source was a series of eight bands of approximately equal intensity and light distribution. This peculiar appearance came on gradually after the lamp was lighted and while it suggested an inversion of the main line it also suggested an equi-partition of the energy of the line into its eight constituents. Janicki[1] who also observed this phenomenon with a Hæreus lamp, states that he found that the line 5461 ÅU finally broke up into five equidistant bands of equal intensity.

Fig. 1.

PHYSICAL LABORATORY,
UNIVERSITY OF TORONTO,
December 1, 1910.

ON A VARIATION IN THE INTENSITY OF THE PENETRATING RADIATION AT THE EARTH'S SURFACE OBSERVED DURING THE PASSAGE OF HALLEY'S COMET.[2]

BY ARTHUR THOMSON, M.A.

SINCE the discovery in 1902 of the penetrating radiation which exists at the surface of the earth, a number of investigators have attempted to ascertain whether the radiation had its origin in the rocks and soil of the earth or in the atmosphere of the latter, or whether it had its origin in one or more of the extra terrestrial bodies such as the sun or the moon.

A number of investigators including Borgmann,[3] Mach and Rimmer,[4] Wood and Campbell,[5] McKeon,[6] Strong,[7] Pacini[8] and Wulf[9] found in their

[1] Ann. der Physik, 19, p. 36, 1906.

[2] Abstract of a paper presented at the Minneapolis meeting of the Physical Society, December 28–31, 1910.

[3] Borgmann, Sc. Abstracts, 1905, no. 1580.

[4] Mach and Rimmer, Phys. Zeit., 7, p. 617, Sept. 15, 1906.

[5] Wood and Campbell, Phil Mag., February, 1907.

[6] McKeon, PHYS. REV., 1907.

[7] Strong, PHYS. REV., July, 1908.

[8] Pacini, Rend. All. Lincei, 18, 123–129, 1909.

[9] Wulf, Phys. Zeit., 10, 1909, pp. 152–157.

various localities daily variations in the intensity of this radiation, and were able from their results to point to regular daily periods of maximum and minimum intensity.

Further, an examination of the results of these investigators shows that the variation in the intensity of the radiation covers an exceedingly wide range.

In some cases variations of ten per cent. from the normal intensity were observed, but in the majority of cases variations very much greater than this amount were commonly noted.

From a closer examination of the various results recorded, however, it does not appear that it is possible to establish any connection between the times of maxima and minima intensities obtained in different localities and as a consequence one is forced to the conclusion that the variations observed by them were due to local causes and not to any influences arising from the proximity of the sun or any of the other celestial bodies.

The results obtained by Professor McLennan and his students at Toronto when investigating this penetrating radiation have been quite different from those obtained elsewhere, in that they failed to show the marked variations observed by others. In particular C. S. Wright who made a number of observations at Toronto was not able to find any appreciable regular variation in the intensity of the penetrating radiation, and the variations which he did observe did not exceed three per cent. of the observed normal intensity of the radiation.

These observations were confirmed later by G. A. Cline and the conclusion was drawn by both these investigators that the soil contributed by far the greater proportion of the penetrating radiation present at the earth's surface at Toronto, and that any portion which might be contributed by the earth's atmosphere or by the sun was negligible in comparison with that arising from the soil.

The view that the soil was the chief source of the radiation was corroborated later by the discovery of Professor McLennan and C. S. Wright that the intensity of the radiation was very much less over the surface of Lake Ontario than over the surface of the land adjoining the waters of the lake.

This diminution in the intensity of the radiation over lake waters was also observed later by Gockel at Vierwaldstättersee in Switzerland.

From these experiments it seems then that there is no doubt about the soil and rocks of the earth being the chief source of the radiation. It appears too that the atmosphere of the earth contributes but a small proportion, if any, of the radiation. There is, moreover, no evidence to show that any part of it has its origin in either the sun or the moon.

Although it is clear from the evidence adduced above that it is not possible to ascribe an appreciable proportion of the penetrating radiation as ordinarily observed to a celestial origin, it seemed possible that the intensity of the

radiation might be increased by the passage near to the earth of a body possessing the constitution and characteristics of Halley's Comet.

And at the suggestion of Professor McLennan the writer undertook to make a series of observations on the intensity of the radiation extending over a number of days during the period when the comet and its tail was predicted to be in closest proximity to the earth.

To make the observations an ionization vessel, of about 30,000 c.c. capacity, similar to that used by C. S. Wright, and later by G. A. Cline, was installed on the roof of the physical laboratory, and readings on the conductivity of the air in the vessel were taken with a sensitive quadrant electrometer situated in a room directly below the point where the ionization vessel was installed.

In setting up the apparatus the most extreme care was taken to see that all insulating parts were clean and dry, that the receiver itself was hermetically sealed and that all electrical connections were effectively and thoroughly screened.

The observations were begun on the morning of May 18, and were continued without interruption until noon on May 21. As a result of these observations it was found that while the normal ionization was about 30 ions per c.c. per second two rather remarkable increases were noted in the observed ionizations. One of these occurred on May 19, beginning at about one o'clock P.M.[1] and lasting until about four o'clock P.M., and the other on May 21, commencing at about half-past nine in the morning and continuing for a period of about one hour and a half. On these two occasions it was found that the ionization rose to about 100 and 130 ions per c.c. per second respectively.

Whether these unusual increases in the ionization were due to the presence of the comet or not, it appears to the writer impossible to decide. A.Wigand,[2] in a paper on observations made at Halle from May 17 to May 20, records decided diminutions in the earth's horizontal magnetic field during the early hours of May 19 and again from 7.36 to 11.30 A.M. of the same day. He also records a marked increase in the electrical conductivity of the atmospheric air between the hours of five and nine o'clock on the morning of May 19 and at the same time an anomalous depression in the potential gradient. Wigand concludes from his observations that these magnetic and electrical disturbances were in all probability connected with the passage of the comet.

The disturbances noted by Wigand it will be seen took place on May 19 at 11 A.M., about twelve hours before the one at Toronto occurred. The phenomena observed by him were the electrical "dissipation" and the potential gradient, while that investigated by the writer was the ionization

[1] Throughout this paper the times refer to 75th meridian mean time.

[2] Wigand, Deutsch. Phys. Ges. Verh., 12, 13, pp. 511–530, July 15, 1910.

in the gas within an hermetically closed metallic vessel. It is possible to conceive that three such phenomena could be related but in the present case it is difficult to see any connection between the two sets of disturbances noted and still more difficult to establish a connection between them and the passage of the comet.

To the writer there appear to be four possible explanations of the disturbance noted by him. It could in the first place be caused by bringing radioactive bodies into the neighborhood of the ionizing chamber. It might also be produced by temporary faulty instrumental conditions or adjustments. Then again the disturbance might be the result of some intense temporary solar activity or finally it might be due to the passage of the comet. The first explanation can be easily dismissed for no radioactive bodies were brought near at any time while the observations were being taken. In regard to the second explanation it may be stated that extreme care was taken to perfect the conditions of measurement. Even after the measurements were finished the whole instrumental outfit was again closely scrutinized, but a rigid examination failed to reveal any faulty adjustment.

The writer is therefore inclined to the view that the disturbance was not due to faulty conditions of experiment or to faulty adjustments.

With the object of corroborating this view observations were again made for periods of about twelve hours on May 23, 24, 25 and 26. These observations failed to show anything abnormal. The ionization varied between narrow limits and the observations were practically of the same character as those made by Wright and Cline on previous occasions.

If the disturbances noted were due to faulty adjustments or to some cause associated with the sun's activity one would expect a repetition of the disturbance. None such occurred, however, during any of the four later periods of observation, and this absence of a repetition would go to confirm the view that the disturbance was not due to any defect in the measuring apparatus.

The absence of a repetition on the four days mentioned, however, would not entirely exclude solar activity as a cause of the disturbance. To establish this point it would be necessary to make a more extended series of observations.

All the readings recorded in the present paper were taken visually, but the strain of making measurements in this manner over long-continued periods of time is too great for visual observation to be practicable, and the writer therefore hopes to continue, at an early date, the observations with self-recording apparatus with a view to seeing whether solar activity is ever really accompanied by such terrestrial disturbances as those noted or not.

This information would be useful in deciding between solar activity and the comet's influence as the cause of the disturbance.

In conclusion the writer wishes to acknowledge his indebtedness to Professor McLennan for suggestions during the investigation, and for his kindness

in scrutinizing the installation of the apparatus, and also to Mr. W. T. Kennedy for his kind assistance in taking some of the readings.

PHYSICAL LABORATORY,
UNIVERSITY OF TORONTO.
Dec. 1, 1910.

AMERICAN PHYSICAL SOCIETY.

REPORT OF TREASURER, DECEMBER 1, 1910.

Receipts.

Dues for 1909	$ 18.00
Dues for 1910	3,339.00
Payments for Section "B," Science Abstracts	71.00
Entrance fees	45.00
Interest on deposit and exchange	25.91
Payments on old accounts	94.00
Deposit in bank, December 1, 1909	451.27
	$4,044.18

Disbursements.

Mileage of president	$ 90.36
Office of secretary: allowance	100.00
Stationery and printing	140.45
Incidentals	6.00
Mileage	79.84
Office of treasurer: allowance	50.00
Stationery and printing	63.00
Incidentals	1.25
Subscriptions to PHYSICAL REVIEW	1,744.50
Subscriptions to Science Abstracts "A"	863.25
Subscriptions to Science Abstracts "B"	72.00
Deposit in bank, December 1,1910	833.53
	$4,044.18

J. S. AMES,
Treasurer.

I certify that I have audited these accounts and that I have found them correct.

WM. J. A. BLISS.

December 12, 1910.

NEW BOOKS.

Die Verwertung des Luftstickstoffs mit Hilfe des elektrischen Flammenbogens. By J. ZENNECK. Leipzig: S. Hirzel, 1911. Pp. 1 + 29. Price, M. 1.60.

A History of the Cavendish Laboratory 1871–1910. New York: Longmans, Green and Co., 1910. Pp. x + 342.

Lehrbuch der Praktischen Physik. By FRIEDRICH KOHLRAUSCH. Leipzig: Teubner, 1910. Pp. xxxii + 736. Price, M. 11.

Die Mechanik, Eine Einführung mit Einem Metaphysischen Nachwort. By LUDWIG TESAR. Leipzig: Teubner, 1909. Pp. xii + 220.

Die Wissenschaft. Sammlung naturwissenschaftlicher und mathematischer Monographien. Grundlagen der Praktischen Metronomie. By KARL SCHEEL. Braunschweig: Vieweg and Son, 1911. Pp. xii + 168. Price, M. 5.20.

Principles of Electrical Engineering. By HAROLD PENDER. New York: McGraw-Hill Book Company, 1910. Pp. xi + 340. Price, $4.00 net.

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An Elementary Text-Book of Physics. Part II. Sound. By R. WALLACE STEWART. London: Charles Griffin Company; Philadelphia: J. B. Lippincott Co., 1909. Pp. 1 + 141. Price, $1.25.

An Elementary Text-Book of Physics. Part III. Light. By R. WALLACE STEWART. London: Charles Griffin and Company; Philadelphia: J. B. Lippincott Co., 1909. Pp. vi + 219. Price, $1.50.

An Elementary Text-Book of Physics. Part IV. Heat. By R. WALLACE STEWART. London: Charles Griffin and Company; Philadelphia: J. B. Lippincott Co., 1910. Pp. 1 + 246. Price, $1.50.

A Text-Book of General Physics for Colleges. Mechanics and Heat. By J. A. CULLER. Philadelphia: J. B. Lippincott Company, 1909. Pp. ix + 311. Price, $1.80.

Traité de Radioactivité. By MADAME P. CURIE. Tome I. Paris: Gauthier-Villars, Imprimeur-Libraire, 1910. Pp. xiii + 426.